SEATTLE REGIONAL MEETING—NOVEMBER 5-6, 1957

JUNE 1957
No. 12
VOL. 28

JOURNAL
OF THE AMERICAN CONCRETE INSTITUTE

THIS MONTH

Papers and Reports.....1105-1168

Form Construction Practices.....
..SUBCOMMITTEE II, ACI COMMITTEE 622 1105

Plastic Hinging at the Intersection of Beams and Columns.....G. C. ERNST 1119

Mass Concrete Control in Detroit Dam.....
.....ROY R. CLARK 1145

Current Reviews.....1169-1176

Discussion.....1177-1284

Discussion contents listed on back cover

News Letter..... 1-42

ACI
PROCEEDINGS
VOL. 53

Stay out of this BOX . . .

"ACI JOURNAL had a paper on the subject that could answer my question. But, I've lost that issue!"

Keep a BOUND VOLUME of ACI Journals on hand.

Immediate reference to expert literature on concrete.

Altogether, in one handsome, durable book, you can have the complete contents of all ACI JOURNAL's published in a one-year period. All ACI technical papers, including the standards, recommended practices, and committee reports which appeared in ACI JOURNAL during that year are presented, with an index to authors and subjects. The year's "Current Reviews" are also indexed.

Bound in blue cloth, lettered in gold, the volume will be an attractive and useful addition to your own, or your firm's technical library.

PRICE TO ACI MEMBERS: $6.00*

(to nonmembers $12.00)

***Bound volumes** 1 to 52 are for sale as far as available at prices to be had on inquiry of the Secretary-Treasurer.

AMERICAN CONCRETE INSTITUTE

PUBLICATIONS

P.O. Box 4754, Redford Station Detroit 19, Mich.

JUNE 1957

No. 12

Vol. 28

PROCEEDINGS VOL. 53

JOURNAL
OF THE
AMERICAN CONCRETE INSTITUTE

Published by the American Concrete Institute under the authority of its Board of Direction

President

WALTER H. PRICE

Vice-Presidents

DOUGLAS McHENRY — PHIL M. FERGUSON

Directors

WILLIAM H. ARMSTRONG — J. W. KELLY
ROBERT F. BLANKS — THOMAS B. KENNEDY
CLAYTON L. DAVIS — WALTER J. McCOY
HARRY C. DELZELL — RAYMOND C. REESE
BRUCE E. FOSTER — LEWIS H. TUTHILL
ROBERT C. JOHNSON — C. D. WAILES, JR.

Past Presidents

C. H. SCHOLER
CHARLES S. WHITNEY — FRANK KEREKES

Cost per copy $1.50. Extra copies to ACI members $1.00.

Papers, committee reports, and other contributions to the **Proceedings** pages of this Journal, as submitted in furtherance of Institute objectives, are, if published, accepted on the sole responsibility of their authors. Institute authority attaches only to standards adopted as provided in the By-laws.

Advertising. See statement under "Alphabetical List of Advertisers," final page **News Letter section.**

Membership dues and Journal subscription rates. Members receive the Journal as one of the considerations of dues payment. Membership grades are: Individual (U. S., its possessions, and North and Central America), $15, Individual (all other foreign countries), $12, Corporation, $50, Contributing, $100, Junior, $7.50, Student, $5. Subscription prices are $12 per year, payable in advance ($15 per year outside North America and U. S. possessions). The Journal is issued monthly, July to June (the volume year being completed by title page, contents, indexes, and closing discussion in Part 2 of the subsequent December issue).

Bound volumes 1 to 52 of PROCEEDINGS of the AMERICAN CONCRETE INSTITUTE (1905 to 1955) are for sale as far as available at prices to be had on inquiry of the Secretary-Treasurer. Special prices apply for members ordering bound volumes in addition to the monthly Journal.

Address: American Concrete Institute, P.O. Box 4754, Redford Station, Detroit 19, Mich. **Copyright, 1957,** American Concrete Institute. Printed In U. S. A. Entered in the Post Office at Detroit, Mich., as mail of the second class under provisions of the Act of Mar. 3, 1879.

Secretary-Treasurer
WILLIAM A. MAPLES

Technical Director
PAUL F. RICE

Managing Editor
ROBERT E. WILDE

Advertising Manager
ANDREW M. SMITH

Manager, Publication Sales
ISABELLE DAVIS

Membership Secretary
EVELYN SCHLITZ

This is to Certify that

John Smith

is a

Member

of this institute and fully entitled to the privileges granted by its Charter and Bylaws

_____
President

_____
Secretary-Treasurer

Date of membership: July 1, 1952

Above is shown the membership certificate now available to all interested members. (Student members become eligible for certificates when they advance to one of the higher membership grades.) The actual certificate is 10 x 12 in. on parchment paper suitable for framing.

Return this order form together with your remittance—to avoid the added expense of billing—and your certificate will be mailed to you. Please allow 90 days for preparation of the certificate.

Please accept my order for a membership certificate at the price of $2.00.

☐ Check ☐ M.O. Enclosed

My name is to appear exactly as follows:

_____
(Print or type)

Signature_____

the ACI JOURNAL

is edited by the Secretary of the Technical Activities Committee under the direction of the Committee

WALTER J. McCOY
Chairman

WALTER H. PRICE
(ex officio)

WILLIAM A. MAPLES
Secretary

PHIL M. FERGUSON

CLYDE E. KESLER

BRYANT MATHER

MICHAEL V. PREGNOFF

L. H. TUTHILL

I. L. TYLER

WILLIAM A. MAPLES
Editor

ROBERT E. WILDE
Managing Editor

MARY K. HURD
Associate Editor

MARIAN PATTEN
Editorial Assistant

RUTH GABLE
Editorial Secretary

Contributions to the JOURNAL

Technical pages of the ACI JOURNAL offer a medium for reporting what is new and interesting in concrete research, design, construction, maintenance, or manufacturing. ACI members and JOURNAL readers are urged to share their knowledge and experience by submitting papers and reports for publication in the ACI JOURNAL. Time and changing interests demand that new contributors with new ideas take their place beside the old.

●

The Technical Activities Committee selects the papers, committee reports, discussions, and other contributions for JOURNAL publication. A volunteer corps of experts in various segments of the field assists in technical appraisal of the manuscripts. Technically speaking, acceptable contributions present original material for improvement of design, manufacturing, or construction; confirm, revise, or upset established ideas or practices; or review, digest, or arrange accumulated experience for more ready use.

Further questions raised by TAC: will the contribution fit into a balanced publication schedule? Can it be published at reasonable expense to the Institute?

●

Contributors should furnish glossy prints or inked tracings of illustrations along with manuscripts submitted in triplicate for publication. Details of the form and physical arrangement of contributions are given in the 1955 *ACI Directory*, p. 14. Separate copies of this "publications policy" are available free on request to the Secretary.

Manuscripts of papers, discussions, and reports should be sent in triplicate to:

Secretary, Technical Activities Committee
AMERICAN CONCRETE INSTITUTE
P. O. BOX 4754, REDFORD STATION, DETROIT 19, MICHIGAN

In ACI Journal next month

The new ACI standard "Recommended Practice for Evaluation of Compression Test Results of Field Concrete (ACI 214-57)" will be published in the July issue. The standard discusses numerous variations in concrete strength, and presents statistical methods which are useful in interpreting these variations. Criteria are offered that can be used to establish specifications and maintain required uniformity.

★

HERBERT A. SAWYER and JACK E. STEPHENS have prepared "Under-Reinforced Concrete Beams Under Long-Term Loads." The effect of an increase in the time of application of load on both deformations and strength of under-reinforced concrete beams is reported. Test results show that increased term of loading causes important changes in moment-curvature relationship.

★

"General Method for Analysis of Flat Slabs and Plates" by JOHN F. BROTCHIE offers a theory for analysis of moments, shears, and deflections in plates. A method is developed for its application to flat slab and flat plate structures. This theory is applicable to any shape internal panels with known column reactions, and tables are presented for the quick determination of moment, shear, and deflection at any point of the panel.

★

WARRINGTON G. MITCHELL'S paper, "Significant Effects of a Waterproof Coating on Concrete Durability," sets forth results of research on impervious surface coatings to maintain concrete durability. Neoprene-latex-coated concrete was subjected to freeze-thaw and sand-blast tests.

★

"Control Joints Regulate Effect of Volume Change in Concrete Masonry" represents the combined efforts of G. A. MANSFIELD, C. A. SIRRINE and BENJAMIN WILK. Practical and easy methods of control jointing are set forth. Authors explain that undesirable fractures in concrete masonry occur rarely, if at all, when good construction practice, embodying an effective control joint procedure is followed in conjunction with structurally adequate designs and proper architectural details.

Title No. 53-62

Form Construction Practices*

Reported by Subcommittee II, ACI Committee 622

JOSEPH R. PROCTOR, JR.
Chairman

N. L. DOE
ROBERT C. JOHNSON
J. C. McCLOSKEY, JR.
O. G. SHARRAR

SYNOPSIS

Subcommittee II, ACI Committee 622 reports on existing practice in construction, design, and use of forms, based on replies to a questionnaire submitted to a selected group of contractors in the United States and Canada. Areas are noted in which contractor efficiency may be improved; architect- or engineer-contractor relationships are treated. Appendix gives detailed summary of answers to questionnaire.

INTRODUCTION

Committee 622—Formwork for Concrete—appointed nine subcommittees for the purpose of thoroughly studying and reporting on literature and actual practices in the design, construction, and inspection of formwork. One objective of the committee is the publication of a recommended good practice for concrete formwork. It is felt that a working guide to safe and efficient formwork is needed in the industry in order to raise the standard of workmanship and thus offset the danger of more stringent controls of formwork methods both from governmental bodies and contract specifications. Subcommittee II is to report on construction practices to determine what good practices are in effect at this time and to advise wherein the general standards can be raised throughout the industry to promote safety, economy, and good workmanship. To accomplish such a purpose, it was felt by the subcommittee that a comprehensive questionnaire sent out to a group of contractors engaged in all types of concrete construction would be invaluable in determining what good practices were in use and in what areas improvement is needed. Obvious bad practices were all too evident in the industry—what was needed was an insight into the operations of the well-established concerns, large and small, to see to what extent formwork had been systematized or standardized, and what degree of thoroughness in planning and design of forms was practical.

*Received by the Institute Mar. 29, 1957. Title No. 53-62 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Sept. 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

Prepared as a part of the work of ACI Committee 622, Formwork for Concrete. The report was submitted to the main committee (17 members) with 12 voting affirmatively and one negatively.

Of corollary importance, the questionnaire was designed to evoke discussion on the details of the architect-contractor relationship in the industry and to encourage suggestions as to how such relations could be improved or enlarged upon. For this reason several of the questions were intentionally worded to discourage a simple "yes" or "no" answer. The questionnaire was mailed to selected contractors in the United States and Canada who, while diversified in types of concrete work performed, nevertheless had in common high standards of workmanship and ability, and preference in handling large and difficult projects.

The questionnaire* was divided into eight parts. Part 1 was devoted to defining, in general, the type and method of forming most prevalent. The function of Parts 2, and 4-7 was to bring out actual practices of the contractors queried. Part 3 was included to picture the relationship between architect-engineer and contractor on the subject of formwork—to show in what areas their specifications affected the construction of formwork and wherein lay any weaknesses in such relations. Part 8 contained general questions pertaining to specifications and performance. In most cases the individual comments proved more enlightening than a cold analysis of the percentage answers.

GENERAL FORM PRACTICE

Formwork has become more and more a significant factor in concrete costs. Almost all concrete structures, with the possible exception of small footings and house foundations, come under examination for possible savings by use of standard reusable panels or clever shoring methods. Even forms used for the encasement of structural steel in concrete may be planned for maximum reusability. Reuse of forms results in savings in materials and ease of handling, but requires careful planning and design to assure necessary wearability to prevent expensive repair and rebuilding. Wherever possible, contractors fabricate their forms in a centrally located shop either on the job site or in their own yard. This enables them to use the most efficient production techniques and to centralize manpower and material to be employed. Where power equipment is available, the size of the form is selected with such handling equipment in mind although small man-handled patented form panels are gaining widespread use in some fields.

The use of steel forms seems to be relatively rare, being largely confined to mass concrete jobs, forms for precast concrete, and some types of patented form panels. One prime disadvantage is the difficulty of attaching inserts and blockouts. Use of steel forms is still prohibited by some specifications due to an unsatisfactory finish frequently obtained, and unless many reuses are possible, the first cost is usually prohibitive. Use of steel for the structure of large forms subject to considerable reuse is far more widespread.

Answers given to questions relating to formwork details give some idea of the degree to which certain methods are practiced. Some of these details

*Tabulation of replies to entire questionnaire is given in appendix to this paper.

however relate most specifically to architectural concrete and would be unnecessary for heavy construction or for concrete structures to be covered by backfill.

There exists considerable variation as to wherein lies the responsibility for the construction of formwork. Forty percent indicated the responsibility lay with the superintendent; 25 percent, with the carpenter or concrete foreman; 17 percent, with the company engineer; and 18 percent, with the owner's representatives. The answers may well have varied had those answering all been considering the same size project. From the individual answers it appears that there is very little definite fixing of responsibility.

Company policies in form building

The existence of company policy in form building and design is a point of great interest to the committee. In only a few cases is a general policy in use, and practically no companies indicated that a formal policy, in the form of a manual or some written instructions, was in effect. It is likely that some companies had more of a policy than they thought. Central office planning frequently results in an informal policy, in that the same individuals usually make up the plans and designs, or rely heavily on methods tried and tested on other jobs. However, this form of policy is dangerous since these individuals have other responsibilities and may be unable, in many cases, to follow up on their form designs to see that they are carried out in the field, or even to judge whether they have been successful or not. Similarly in regard to form details such as method of joining form panels together, method of securing wales to form panels or bulkhead details, most answers indicated these were detailed separately for each job, but no doubt some continuity exists where the same individual makes the design for several jobs. Here again, such continuity is unreliable.

There appears an almost equal split between those in which design responsibility remained with the main office and those in which designs and plans were made on the job. It would seem that complete standardization of design policy would necessitate locating such responsibility centrally. However, with the existence of an established formal policy, such standardization is equally possible with the designs being made right on the project. In either case the listing and ordering of form materials should be performed by someone familiar with the specific design and general policy, if any, regarding grade of material and overall material needs for the project.

Form watchers

The majority of contractors indicated that no specified number of form watchers was insisted upon. Some answers indicated that form watchers were provided as required by circumstances; no doubt in most cases this amply covers requirements. However, this may well be an area in which some definite policy is essential. Further, it would be in order at the same time to prevent undue burdening of form watchers with additional carpenter

work other than checking alignment, watching and adjusting wedges and shoring, and other jobs. Such time-consuming additional duties frequently assigned to form watchers are closing washout holes, placing waterstops, keyways, templates, erecting scaffolds and walkways for concrete crews, and other functions normally associated with preparing the form for concreting.

CONSIDERATION OF SAFETY IN FORMWORK

Little uniformity in safety procedures exists among contractors. In general, sound, economical construction automatically implies a safe form. However, too frequently, reliance is placed on the discretion and judgment of the carpenter foreman, concrete foreman, or even the owner's inspector—persons who are not directly concerned with this aspect. A more uniform specification of minimum safety precautions would eliminate this difficulty. Furthermore, it would tend to equalize bidding on concrete work by requiring more careless operators to conform to the routine precautions of the better builders. Recognition of and provision for the elimination of unsafe practices in formwork is the surest way of precluding the possibility of stricter building laws. Such controls come about usually as a result of accidents involving injury, death, or property damage caused by alleged negligence of the constructor. The Associated General Contractors of America provides its members with general construction safety standards; reference to the section on formwork therein would improve most architectural specifications and inspection practice.

The most serious accidents during construction of concrete buildings seem to have been associated with, if not primarily caused by, construction loads imposed upon recently cast concrete. While proper safety precautions vary with conditions of each project, and detailed specification is manifestly impracticable, a general specification of minimum safety precautions is possible. Committee 622 invites comment on the following alternatives for such a uniform specification.

1. *When insufficient job-cured test cylinders are available, all construction loads shall be carried to the soil by the original shoring left in place until concrete construction is completed.* This requirement, paraphrased from the German building code, would be suitable for a height limited perhaps to three stories. Construction loads would include fresh concrete, forms, other building materials, construction equipment, and personnel. In no case would the weight of fresh concrete for a higher level be supported by the recently cast concrete of a lower level.

OR

2. *No construction loads exceeding the design load shall be supported upon any portion of the structure under construction. No construction load shall be supported upon, nor any shoring removed from any portion of the structure under construction until that portion of the structure has attained sufficient strength, as demonstrated by tests on job-cured specimens, and as required by a competent structural analysis for the loads proposed and the strength demonstrated. Such*

analyses and test data shall be furnished by the contractor to the engineer for approval before removal of shoring.

This requirement, though more elaborate, would permit usual American construction practice in multistory reinforced concrete building of any height. The analyses and strength data on job-curing would provide a flexible minimum safety precaution. The last statement would even provide against collapse under dead load of the structure itself by insuring adequate curing.

ARCHITECT- OR ENGINEER-CONTRACTOR RELATIONSHIP

Replies to questions pertaining to architect- or engineer-contractor relationships were varied and divergent particularly in regard to the degree of formwork specification. Most seemed to feel that end results only should be specified. Essentially, this is the case at present with most projects, although specifications of form materials, finish, stripping time, and other items frequently occur on some of the larger or governmental jobs. A few specifications require submission of formwork drawings to the engineer for prior approval, although many require prior approval of stripping and reshoring sequences. Even fewer specify in any way the type, manner, or degree of shoring required. In any event, approval by the engineer of various steps does not relieve the contractor of the ultimate responsibility for the appearance, strength, and shape of the finished structure. The question was a bit leading in that it did appeal to the contractor who wished merely to eliminate annoying inspectors. Nevertheless, it is felt that formwork is a contractor's device for constructing and depends to a large degree on his foresight, ingenuity, and materials at hand. Therefore, its design and construction should remain the contractor's province.

Replies were unanimously in favor of designers giving more thought to formwork when designing structures. Reducing the number of irregular shapes, establishing uniform sizes of structural members, careful planning of construction joints, consideration of stock sizes of materials to minimize cutting, consideration of stripping problems, are all ways in which formwork costs may be reduced. To conserve concrete by designing irregular shapes is false economy; the resultant high forming costs offset any such saving. Most contractors felt it reasonable and necessary that engineers specify certain features of formwork as they do for other construction operations. Such requirements might be tolerances for line and grade, finish required, adherence to construction joints shown on plans except by special approval, camber where necessary, minimum stripping times to safeguard structure, and use of such details as chamfers which tend to improve the appearance of the structure. They did feel that tolerances, finishes, and stripping times should be so specified as not to cause unreasonable expense. Frequently, a finish is specified which cannot be obtained with the material specified for the form. In this case, merely specifying the finish should be adequate, leaving the method of attaining it to the contractor. Elimination of cutting forms for waterstops wherever possible would constitute considerable savings.

In some cases, engineers write into their specifications a stock formwork specification such as the one published by Portland Cement Association for architectural concrete. In such cases if exception is not made for structures to be backfilled or otherwise not requiring architectural treatment, considerable unnecessary expense is incurred. In such cases, field inspection frequently overlooks the more stringent requirements as unnecessary which tends to cause the contractor to take a cynical view toward the specifications as a whole.

CONCLUSION

The questionnaire served its purpose well in pointing out several areas in which considerable progress may be made by contractors in improving efficiency. Furthermore, it brought into focus some of the problems involved within the framework of the architect-engineer-contractor relationship. It will remain for this subcommittee to make its recommendations for preferred field practices for incorporation into the final overall committee report. It is hoped in the meantime, however, that the results of this questionnaire may be of assistance to the other subcommittees of Committee 622 by providing a measure of the problems and conditions of existing construction practices. Growing use of patented form panels and of mass production of formwork elements, the increased realization of architects and engineers of the value of concrete design with formwork costs in mind, and the increased interest in formwork safety on the part of governmental bodies due to recent tragic form failures have focused considerable attention on general formwork practices both in design and construction. The relative lack of up-to-date literature on the subject only increases the need for some sort of code or guide to promote more systematization or uniformity in this field. By increasing the awareness of the problems and challenges of formwork as a constructor's tool, such a code or guide would make an outstanding contribution to the industry.

APPENDIX

Summary of answers to questionnaire on formwork received from selected group of contractors in the United States and Canada is presented below.

1. Nature of formwork

Does your work consist mostly of large areas that call for a number of similar placements where reuse is a dominant factor?

Reuse is usually dominant—74 percent
Reuse not dominant—26 percent

Is shoring or posting a significant part of your forming operation, or is the emphasis on the form itself?

Shoring is a significant part—60 percent
Shoring not a significant part—40 percent
Shoring significant for bridges only—4 percent

Is much of your formwork hung from steel framing?

Much—20 percent
Not much—80 percent
Bridges only—2 percent

Does your average job permit the use of large forms, which can be handled by power equipment, or do you tend to utilize smaller man-handled units?

Forms are usually power handled where feasible—60 percent
Forms are seldom if ever power handled—40 percent

Do you find it more practical to fabricate "in place" or do you utilize shop fabricated or patented panels?

Fabrication in place is the rule—35 percent
Shop fabrication or prefabricated panels are the rule—65 percent

Do you frequently use patented form panels or systems and, if so, what brand and for what type structures?

Patented form systems are regularly used—40 percent
Patented form systems used seldom if ever—60 percent

Do you occasionally or frequently use steel forms?

Steel forms are regularly used—33 percent
Steel forms seldom if ever used—67 percent

Do you work predominantly with owners or agencies which have definite specifications regarding formwork? If so, what requirements do you find most restricting or annoying? Which do you feel most necessary?

Work is predominantly under some formwork specifications—48 percent
Work not under formwork specifications—52 percent

Requirements felt necessary	*Requirements found restrictive or annoying*
Specifications wholly satisfactory	Vague on finish required
None	Specification of *method* and *result*
Architect's specifications are good	Stripping time
Specify T&G or plywood; V-strips at joints	Do not consider lumber, plywood sizes
Engineer's approval of form design	Special linings not necessary
Exposed work—specify material	Specifications on surface and lining—increase expense but not value
Mortar tight	Prohibition of steel forms
Metal forms sufficiently stiff	Too close tolerances
Bracing and shoring requirements	Plywood form on backfill covered work
Chamfer strip for corners	Restriction on use of metal form
Reposting	
Finish required	
Chamfer, lines, tightness	
Physical condition of panels	

2. Company policy

Do you have a standard company policy on form planning and design?

Yes—29 percent
No—71 percent

Is the policy written down in a manual or bulletin of some kind? If so, how comprehensive is it?

Policy written—8 percent
Not written—92 percent

Do you detail the forms for each job, or do you select details from your standards as they apply to a job?

Forms are detailed for each job—92 percent
Standard details are available—8 percent

Do form drawings specify such points as rate of concreting, temperature, type, and grade of material?

	Form drawings specify	Do not specify
Rate of concreting:	46 percent	54 percent
Temperature of mix:	36 percent	64 percent
Slump or mix:	33 percent	67 percent

Are shoring and scaffolding subject to standard design policy?

Yes—25 percent
No—75 percent

Where does the responsibility for form design lie within the organization?

Main office—43 percent
On the job—57 percent

Who lists and orders form materials?

Engineer—67 percent
Purchasing agent—15 percent
Carpenter foreman—22 percent
Superintendent or other—12 percent

(Some overlap included)

Do you require a specified number of form watchers?

Yes—34 percent
No—66 percent

Do you have different classifications for formwork in order to specify standard design procedures? If so, what are these classifications?

Classes of formwork under standard policy—15 percent
Classes of formwork not under standard policy—85 percent

Do you specify in form drawings or in standards of form construction the size nails to be used for specific purposes, i.e., nailing of inserts, nailing shoring, etc.?

Yes—26 percent
No—74 percent

How much safety scaffolding is built into forms for high lifts?

Safety scaffold built into forms on high lifts—61 percent
Little or none—35 percent
Separate from forms—4 percent

3. Relations with architects and engineers

Do architects, engineers, or other owner representatives with whom you customarily work require submission of form drawings for prior approval?

Usually required—19 percent
Sometimes required—25 percent
Never required—56 percent

Do they specify or require approval on stripping and reshoring sequences?

Usually required—84 percent
Not required—16 percent

In what general area do their criticisms, if any, usually lie? (tie spacing, shoring, size of members, etc.)

Tie spacing—5 percent
Surface appearance—10 percent
Shoring and reshoring—10 percent
Exceeding tolerances on line and plumb—2 percent
Time of stripping—14 percent
Cleanliness—0 percent
Spacing lifts—2 percent
Tightness—0 percent
Stud and waler spacing—2 percent
Little or no criticism—55 percent

How do they restrict your formwork?

Methods—24 percent
Use of materials—42 percent
Time of stripping—94 percent
Re-posting of slabs—82 percent

Are you usually forewarned of these restrictions by specific mention in the specifications, or do they develop in the course of inspection?

Restrictive requirements are clearly stated in specifications—76 percent
They develop with no advance warning as the inspectors choose to interpret the specifications—24 percent

4. Planning and design

Do your form plans and design make provisions for the following?

Form design and plans make provision for:
Rate of concreting—67 percent
Temperature of concrete—45 percent
Vibration—65 percent
Handling facilities—65 percent
Method of placing—69 percent
Height of drop—60 percent
Bulkheads—60 percent
Shoring and posting—67 percent
Reshoring—47 percent
Stripping procedures—65 percent
Camber—56 percent
Assumption of loads by concrete while stripping—36 percent

Of the following minor details of formwork, check whether details are worked into the form design or whether they are left to the carpenter foreman's discretion.

	Part of form design	Left to carpenter foreman
Bulkheads	39 percent	61 percent
Joining form panels	51 percent	49 percent
Scaffolds	36 percent	64 percent
Anchor bolt templates	45 percent	55 percent
Runways for concrete buggies	22 percent	78 percent
Shoring and bracing of posts	68 percent	32 percent
Installation of keyways	42 percent	58 percent
Screeds	22 percent	78 percent
Clean out and inspection holes	24 percent	76 percent
Anchors against uplift	65 percent	35 percent
Provisions to realign during concreting	40 percent	60 percent
Spreaders and their removal	25 percent	75 percent
Splicing of members	37 percent	63 percent
"Wrecking strips" to aid stripping	35 percent	65 percent
Provisions to handle form	44 percent	56 percent

What form pressures (horizontal) do you use for design purposes?

150 lb per cu ft liquid—57 percent
10 ft 0 in. head maximum—3 percent
8 ft 0 in. head maximum—3 percent
6 ft 0 in. head maximum—7 percent
} Others state no maximum

100 lb per cu ft liquid—(no maximum stated)—7 percent
1000 psf maximum up 7 ft 0 in. lifts—3 percent
Columns—1500 psf maximum; walls—1000 psf maximum—3 percent
As recommended by form manufacturer or Portland Cement Association—30 percent

What allowable design stress values do you use for lumber?

2000 psi— 5 percent
1800 psi— 7 percent
1600 psi—12 percent
1400 psi— 3 percent
1200 psi—20 percent
1000 psi—15 percent
As recommended by timber associations—38 percent

What live loading, vertical and lateral, do you use for floor and roof form systems?

VERTICAL

With power buggies: 75 psf—46 percent
50 psf—15 percent
40 psf—15 percent
30 psf— 8 percent
1 ton concentrated—8 percent
Actual weight concentrated—8 percent
30 percent impact—8 percent
100 percent impact—15 percent

With other distribution methods: 80 psf— 6 percent
70 psf— 6 percent
50 psf—41 percent
40 psf—12 percent
37½ psf—6 percent
30 psf—18 percent
25 psf— 6 percent
20 psf— 5 percent

Thin shell: 20 psf—6 percent

LATERAL

None—88 percent
Variable—12 percent

What is the maximum angle at which you place a slab without a top form?

15 deg— 3 percent
20 deg—18 percent
25 deg—15 percent
30 deg—18 percent
35 deg—12 percent
40 deg— 3 percent
45 deg—26 percent
50 deg— 3 percent

No experience—3 percent

Included in other { Thin shells 60 deg—3 percent; Mass concrete 20 deg—3 percent }

Does the finish of concrete surfaces dictate your form needs?

Often—84 percent
Seldom—16 percent

Do you use patented form panels? If so, do you utilize the engineering assistance made available by the manufacturer?

Frequently use—35 percent
Seldom use—65 percent

5. Methods of building formwork

Who is responsible for final inspection of form before concreting?

Our engineer—17 percent
Job or general superintendent—40 percent
Carpenter or concrete foreman—25 percent
Owner's representatives—18 percent

Who sets up procedure for providing means of aligning form before and during concreting, i.e., location of wedges, jacks, etc.?

Job superintendent—38 percent
Carpenter or concrete foreman—60 percent
Our engineer—12 percent

Do you require all wedges to be nailed in final position?

Yes—56 percent
No—44 percent

Do you kerf moldings and other small inserts to facilitate stripping?

Yes—67 percent
No—28 percent
Sometimes—5 percent

What type calking do you use for joints in form panels for architectural concrete?

Metal—3 percent
Wood—3 percent
Putty—9 percent
Masking tape—4 percent
None used—42 percent
Spackle plaster—17 percent
Grease or tallow—9 percent
Mastic—6 percent
Rubber—3 percent
Mortar—4 percent

Do you employ jigs and templates for spacing studs, tie holes, etc., wherever possible?

Usually—69 percent
Never—31 percent

6. Methods of placing

What devices do you use to detect movement of form?

Plumb bob—21 percent
Level and transit—47 percent
Observation of superintendent, foreman, carpenter, or owner's representative—21 percent
Taut lines—47 percent
"Feeler" markers to fixed objects—12 percent
(Total more than 100 percent since some use more than one method.)

How many form watchers do you use?

None—4 percent
As necessary for job conditions, or as required by law, union rules, or owner's representative—35 percent
Have a standard policy requiring a fixed number for different classes of our work—61 percent

Is the method of placing considered in form design to allow for:

Weight of crews and equipment? Yes—95 percent No—5 percent
Movement of buggies and other equipment? Yes—95 percent No—5 percent
Attachment of chutes, hoppers, tremies, side entries? Yes—80 percent No—20 percent

Do you wet forms and inserts thoroughly before concreting?

Forms are wetted (summer only)—85 percent
Forms not wetted—4 percent
Oiled instead—11 percent

7. Methods of stripping

Do you use double headed nails to facilitate stripping?

Yes—75 percent No—25 percent

Do you attempt to isolate odd shapes like corbels, etc., by means of breakaway lines so that main form will strip without damage?

Yes—93 percent No—7 percent

Do you employ crush plates or wrecking strips in tight places to protect form while stripping?

Yes—63 percent No—33 percent Sometimes—4 percent

Do you follow any procedure or rules for stripping time in absence of requirements by architect or engineer?

Yes, we have fixed rules—77 percent
No, responsibility for time of stripping lies with the job superintendent—23 percent

8. General—Regarding specifications

Do you feel architect or engineer specifications should be confined to the finished product rather than construction methods?

Specifications should cover results desired only, with exceptions for special work—88 percent
Specifications should detail methods only—0 percent
Prefer the present combination of methods and results; both specified and inspected—12 percent

Do you feel architects and structural engineers could give more thought to simplifying concrete designs to facilitate reuse of forms without revamping?

Yes—100 percent

Do you have to meet any legal requirements showing construction of shoring and reshoring such as those now in force by the City of New York? (Design of forms by registered engineers, inspection by city inspectors, affidavits certifying construction of forms before placing, etc.)

Yes*—10 percent
No—85 percent
Sometimes—3 percent

Have you experienced any failures of important formwork? If so, can you summarize practices responsible? Were any persons injured? What did failure cost? (Do *not* identify project nor parties involved.) In what year did it occur?

We have had failures in formwork—8 percent
No failures—92 percent

SUMMARY OF DATA ON FAILURES

Injuries	*Causes*	*Cost*	*Year*
Minor	Careless workmanship	—	—
Minor	Poor inspection	$500.00	1956
Minor	Wrong or faulty placement	—	—
Four killed	Steel shoring failed	$100,000.00	1942
Minor	Failure of wall ties	—	—
Minor	Too rapid placing	$300.00	1946
Minor	Form tie failure	—	—
None	Misalignment	Large	1950
Three Injured	Inadequate diagonal bracing	$2,000.00	1954
Minor	Worn out threads on tie bolts	—	—

In what ways do you feel specifications for formwork can be improved or expanded to improve the caliber of concrete work?

1. Include tolerances for various classes of work.
2. Specifications should allow lumber and plywood dimensions for structural members; designs should plan on multiples of 4-ft panels.
3. Stop confusing method *and* results; specify one or the other, preferably results.
4. Specify minimum sheeting thickness for given stud spacing.
5. Specify results.
6. More consideration for practical conditions.
7. Specifications should state stripping times in advance and not leave to job inspector's whim.
8. Architects should not expect plaster-smooth finish from plywood forms; specifications should state finish desired in detail and in a realistic practical fashion not incompatible with form material where this is also specified.
9. No specifications on forms; leave to contractor.
10. Conferences before construction.
11. Include camber.
12. Eliminate sharp corners—V-strip at horizontal construction joints.
13. More and *better* inspection.
14. More details.
15. Allow use of curing compound instead of water cure.

*Includes work done in New York City.

16. Eliminate restrictive conditions, i.e., thin walls, much steel.
17. Eliminate surface finish (bridges) by using textured finish.
18. Specify grade of plywood required when top grade finish is desired.
19. Simple regular design in concrete.
20. Specify field-cured job cylinder strength for stripping instead of given period of time.
21. Architect and/or engineer should "back up specifications he has."
22. Specifications should be general (performance); finish specified.
23. Designers need more field experience.
24. Eliminate unnecessary smoothness in finish specifications.
25. Use cheap ties where appearance is unimportant.
26. Using complicated forms to reduce concrete is false economy.
27. Allow as wide a range of form face materials as possible.
28. Eliminate rubbing which spoils original surface.
29. Eliminate rubber waterstops requiring cutting into forms.
30. Base stripping time on job cylinder tests.
31. Require adequate strength; securing it is contractor's responsibility.
32. Use performance specifications.
33. Require tolerance reasonable for use and commensurate with economy.

Discussion of this paper should reach ACI headquarters in triplicate by Sept. 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-63

Plastic Hinging at the Intersection of Beams and Columns*

By G. C. ERNST†

SYNOPSIS

Principal object of investigation was to determine the amount of concentrated plastic rotation developed at the connection between beams and columns. Thirty-three tests were conducted, some at a slow loading rate, some at a fast loading rate equivalent to that prescribed for concrete cylinders. Steel ratios of 0.01, 0.03 and 0.05, column widths of 6, 12, 18, 24, and 36 in., and nominal concrete strengths of 3000 and 4000 psi were used.

Concentrated plastic rotation at concrete crushing and at maximum moment is markedly reduced when the steel ratio exceeds 0.01, and is also less for a fast loading rate. Theoretical moments agreed satisfactorily with experimentally determined moments in all cases. At concrete crushing for the 0.05 steel ratio under fast loading, concentrated plastic rotation was virtually negligible. It seems necessary, therefore, to consider the effect of steel ratio and rate of loading in cases for which the ultimate capacity of a structure is dependent upon a redistribution of moment produced by concentrated plastic rotations.

INTRODUCTION

Object and scope

Recent theoretical and experimental studies[1,2,3] have provided information on the plastic response of reinforced concrete beams and connections of the type indicated as *A* and *C* in Fig. 1. Such studies have shown the desirability of obtaining quantitative information on the magnitude of the concentrated plastic rotation that develops at an abrupt change in moment which occurs where a beam enters a column (Type *B* connection, Fig. 1) or at a load concentration. An advantageous redistribution of internal moments and shears in statically indeterminate structures is dependent upon an adequate development of plastic rotations, with a resulting maximum of energy-absorbing capacity under extremes of loading.

This investigation concerns only Type *B* connections having various external proportions and internal steel contents; tests were divided into two groups according to the rate of loading. One group consisted of 15 new and three repaired beam-to-column connections tested in the customary manner by stopping at specific increments of load or deflection in order to take strain readings and photographs, with total loading time one hr or more. The second

*Received by the Institute Aug. 24, 1956. Title No. 53-63 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Sept. 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

†Member American Concrete Institute, Professor of Civil Engineering and Director of the Engineering Experiment Station, University of Nebraska, Lincoln, Neb.

group of 15 companion specimens was tested so that the outermost fibers in compression were stressed at 20 psi to 50 psi per sec, as specified for concrete cylinders in ASTM C 39. This fast loading rate required simultaneous recording of load, deflection, and strain by means of oscillographs, but the accumulation of plastic strain is believed to be more nearly representative of that during the test of a standard cylinder.

Notation

L = distance from support to support of beam-to-column specimens, in.

w = width of column stub parallel to L, in.

S = $\frac{L}{2} - \frac{w}{2}$ except when designating the slow loading group

d = distance from outermost compression fiber to centroid of tension reinforcement, in.

d' = distance between centroids of tension and compression reinforcements

b = breadth of beam, in.

k = ratio of the distance between compression face and neutral axis to the distance d

k' = d'/d

f_c' = compressive strength of 6 x 12-in. cylinders, psi

f_c = stress in outermost compression fiber, psi

f_{av} = average compression stress in concrete, psi

k_1k_3 = f_{av}/f_c'

f_y = yield point of tension reinforcement, psi

f_s = stress in tension reinforcement, psi

f_y' = yield point of compression reinforcement, psi

f_s' = stress in compression reinforcement, psi

ϵ_c = unit strain in concrete, in. per in.

ϵ_u = ultimate unit concrete strain in flexure, in. per in.

ϵ_s = unit strain in tension steel, in. per in.

ϵ_s' = unit strain in compression steel, in. per in.

ϵ_y = unit strain at beginning of yield point range, in. per in.

ϵ_{wh} = unit strain at beginning of work hardening, in. per in.

A_s = area of tension steel

A_s' = area of compression steel

p = A_s/bd

p' = A_s'/bd

n = ratio of modulus of elasticity of steel to modulus of elasticity of concrete

I = moment of inertia of transformed section, in.^{4}

M_y = moment at yield of tension reinforcement, in.-lb

M_c = moment at crushing of outermost concrete fibers, in.-lb

M_{max} = moment at maximum load, in.-lb

q' = reinforcing index, $\frac{pf_y - p'f_y}{f_c'}$, of actual beam

q'_{cr} = reinforcing index, $\frac{\epsilon_u}{\epsilon_u + \epsilon_y} k_1k_3$, defining balanced reinforcement

q'_{wh} = reinforcing index, $\frac{\epsilon_u}{\epsilon_u + \epsilon_{wh}} k_1k_3$, defining limit for work hardening

ϕ = unit rotation in radians with subscript y, c, or m to denote value at steel yield, concrete crushing, or maximum load, respectively, and additional subscript p if plastic only

Θ_p = total concentrated plastic rotation, radians

P_c = load at concrete crushing

P_{max} = maximum load

Δ = deflection, with subscript designation as for ϕ, in.

C = crushing of outermost compression fibers, on charts

Y_N = development of yield strains in tension steel at north column face, on charts

Y_S = development of yield strains in tension steel at south column face, on charts

Y_{CL} = development of yield strains in tension steel column at center line, on charts

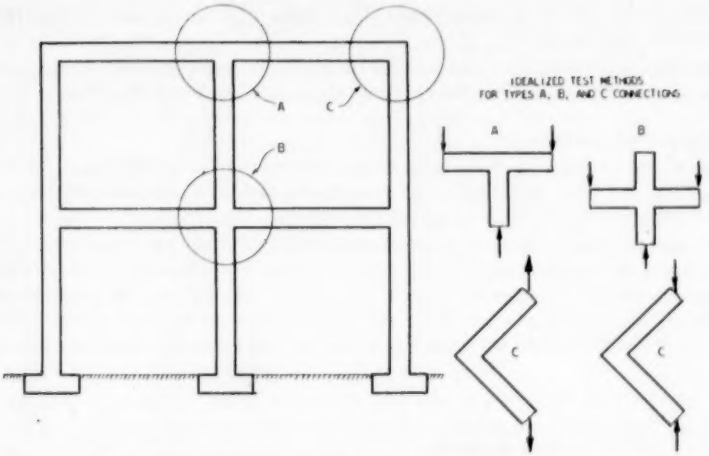


Fig. 1—Typical repetitive connections in reinforced concrete structures

SPECIMENS AND TEST METHOD

Test specimens

All test specimens were 6 x 12 in. in cross section, 10 ft long, with a span of 9 ft. Three groups of ten beams were cast, each group having a different steel ratio. Within each group, column stubs were integrally cast on top and bottom of each beam at midspan, with dimensions ranging from 6 x 6 x 9½ in. to 6 x 36 x 9½ in. as shown in Fig. 2. The load was applied through the column stub.

Each specimen is designated by two numbers and one letter, e.g. 1-6S. The first number refers to the steel ratio group, Group 1 for 0.05, Group 2 for 0.03, and Group 3 for 0.01; the second number provides the column stub width (in.), and the letter identifies the type of loading, S for slow, F for fast.

Stirrups were made of #3 deformed bars bent into the form of column ties and spaced so that all of the predicted maximum shear could be carried solely by the stirrups at a unit stress well below their yield point.

Two #3 longitudinal bars used in the top of each beam to maintain the shape of the reinforcement cages also served as compression reinforcement.

Arrangement of longitudinal and transverse steel and all dimensions are shown in Fig. 2 for each group of beams. Principal properties are listed in Table 1. Pairs of identical beams were made, one to be tested at a slow rate and one at a compression stress rate equal to that specified for concrete cylinders in ASTM C 39.

Materials, fabrication, and curing

The test specimens were cast in three groups of ten, each group cast from a single 2½ cu yd delivery of ready-mixed concrete. The concrete had a nominal 28-day strength of 4000 psi or 3000 psi, with a slump of 3 to 4 in. Average strengths at time of test were:

Group 1S—4180 psi	Group 1F—4820 psi
Group 2S—4170 psi	Group 2F—4710 psi
Group 3S—3300 psi	Group 3F—3230 psi

Each group was cast in less than 30 min, during which time the concrete was vibrated in place with a 20,000 vpm vibrator. Forms for the top column stub were removed after 4 to 6 hr, and the beam troweled to the intersection with the stub.

All beams were kept moist under plastic drop cloths for 2 weeks, and then air dried for 2 weeks or more prior to test.

Intermediate grade deformed bars meeting the requirements of ASTM A 15 and A 305 were used throughout. Properties for these bars are given in Table 2 and Fig. 3.

Test set-up and procedure

All beams were tested on a specially designed support in a 400,000-lb hydraulic testing machine (Fig. 4). The 15 beams of Groups 1S, 2S, and 3S which were tested under slow rates of loading did not utilize the load cells shown.

Steel strain instrumentation consisted of Type PA-3 SR-4 electric resistance gages placed on the outermost bars through core holes after curing was completed. All specimens had these gages at the center line of the column stub, and for the slow loading group all but those with 6 x 6 x 9½-in. stubs also had PA-3 gages just outside of the face of the stub, making two gages for those with the 6 x 6 x 9½-in. stubs and six gages for all others. Those tested

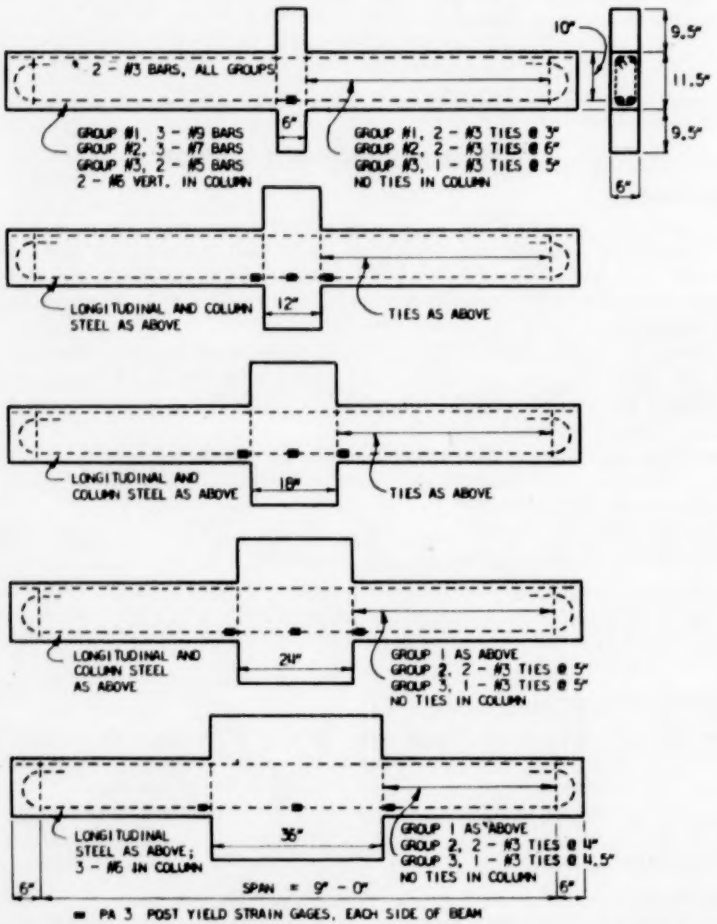

2 - #3 BARS, ALL GROUPS
10"
9.5"
11.5"
9.5"
6"
GROUP #1, 3 - #9 BARS
GROUP #2, 3 - #7 BARS
GROUP #3, 2 - #5 BARS
2 - #6 VERT. IN COLUMN
GROUP #1, 2 - #3 TIES @ 3"
GROUP #2, 2 - #3 TIES @ 6"
GROUP #3, 1 - #3 TIES @ 5"
NO TIES IN COLUMN
6"
LONGITUDINAL AND COLUMN STEEL AS ABOVE
12"
TIES AS ABOVE
LONGITUDINAL AND COLUMN STEEL AS ABOVE
18"
TIES AS ABOVE
LONGITUDINAL AND COLUMN STEEL AS ABOVE
24"
GROUP 1 AS ABOVE
GROUP 2, 2 - #3 TIES @ 5"
GROUP 3, 1 - #3 TIES @ 5"
NO TIES IN COLUMN
LONGITUDINAL STEEL AS ABOVE; 3 - #6 IN COLUMN
36"
GROUP 1 AS ABOVE
GROUP 2, 2 - #3 TIES @ 4"
GROUP 3, 1 - #3 TIES @ 4.5"
NO TIES IN COLUMN
6"
SPAN = 9' - 0"
6"
= PA 3 POST YIELD STRAIN GAGES, EACH SIDE OF BEAM

Fig. 2—Dimensions, steel content, and steel strain locations, S-series

under fast loading had gages on the steel at the center line of the column stub only, for the purpose of determining the extent to which steel yield spread into the column.

Concrete strains were obtained on the beams of Groups 1S and 2S by placing Type A1 SR-4 gages on the vertical faces of the beam, ¾ in. down from the top and ¾ in. away from each column face in the direction of the reactions, four gages on the concrete per beam. The beams of Group 3S had Type A1 SR-4 gages placed on the top surface ¾ in. away from each column face in the direction of the reactions and located midway between the vertical faces, plus the four gages previously described, making six gages on the concrete per beam of Group 3S.

Concrete strains were obtained on Group 1F specimens by placing Type A1 SR-4 gages on the top surface at ¾ in. and 2¾ in. from each column face and located midway between the vertical faces. Group 2F and 3F specimens had Type A1 SR-4 gages placed on the top surface at 1¾ in. from each column face and located midway between the vertical faces. The two locations of the gages on Group 1F showed no significant difference in the load at which concrete crushing was indicated, hence the use of the single arrangement for Groups 2F and 3F. Average strains of the gages were recorded on oscillographs.

Deflections for the S-groups were obtained by an engineer's level sighted at a scale at mid-span of the beams having a 6 x 6 x 9½-in. column stub and at the face of each column stub for all other specimens. With the loading head blocked against rotation by the jacks shown in Fig. 4, deflections at opposite column faces were found to be the same. Support deflections, observed on scales, were found to be negligible.

The center deflections for the F-groups were recorded on an oscillograph by means of a column-type transducer made from clock spring steel with Type A1 SR-4 resistance gages on each side of the point of maximum strain.[4]

The test procedure for Group 1S consisted of taking strain and deflection readings at specific load increments up to failure, with crack inspections at each load increment. A photograph was taken of each beam after the test had been completed. These beams had a steel ratio of 0.05, with virtually no yield or plastic portion on the load-deflection curve.

For Groups 2S and 3S having an appreciable yield portion, load increments were taken only up to initial yielding of the steel, after which strain increments were used and for which

TABLE 1—PROPERTIES OF BEAMS

For all beam groups, d = 10.0 in., d' = 8.5 in., and compression steel consists of two #3 bars.

Group	Specimen No.	Column stub width, in.	Transverse ties	q_{cc}' at P_c*	q_{cc}' at P_{max}†	q_{cs}'†	Mode of failure
Group 1—Three # 9 tension bars	68‡	6	2- # 3 @ 3 in.	0.306	0.846	—	Compression
p = 0.0500	128	12	2- # 3 @ 3 in.	0.357	0.737	—	Compression
p'/p = 0.074	188	18	2- # 3 @ 3 in.	0.372	0.499	—	Compression
q' = 0.489	248	24	2- # 3 @ 3 in.	0.380	0.650	—	Compression
f_c' = 4180 psi	368	36	2- # 3 @ 3 in.	0.362	0.546	—	Compression
Group 2—Three # 7 tension bars	68	6	2- # 3 @ 6 in.	0.605	0.850	0.102	Tension
p = 0.0300	128	12	2- # 3 @ 6 in.	0.793	0.750	0.190	Tension
p'/p = 0.123	188	18	2- # 3 @ 6 in.	0.968	0.868	0.213	Tension
q' = 0.284	248	24	2- # 3 @ 5 in.	0.791	0.889	0.153	Tension
f_c' = 4170 psi	368	36	2- # 3 @ 4 in.	0.876	—	0.212	Tension
Group 3—Two # 5 tension bars	68	6	1- # 3 @ 5 in.	0.600	—	0.126	Tension + work hardening
p = 0.0103	128	12	1- # 3 @ 5 in.	0.581	—	0.129	Tension + work hardening
p'/p = 0.359	188	18	1- # 3 @ 5 in.	0.495	—	0.107	Tension + work hardening
q' = 0.098	248	24	1- # 3 @ 5 in.	0.575	—	0.116	Tension + work hardening
f_c' = 3300 psi	368	36	1- # 3 @ 4.5 in.	0.689	—	0.148	Tension + work hardening

*Computed with ϵ_c at P_c (see Table 4).
†Computed with ϵ_c at P_{max} (see Table 4).
‡Properties for the F-series are same as for S-series, except f_c' = 4820, 4710, and 3230 psi for Groups 1F, 2F, and 3F, respectively.

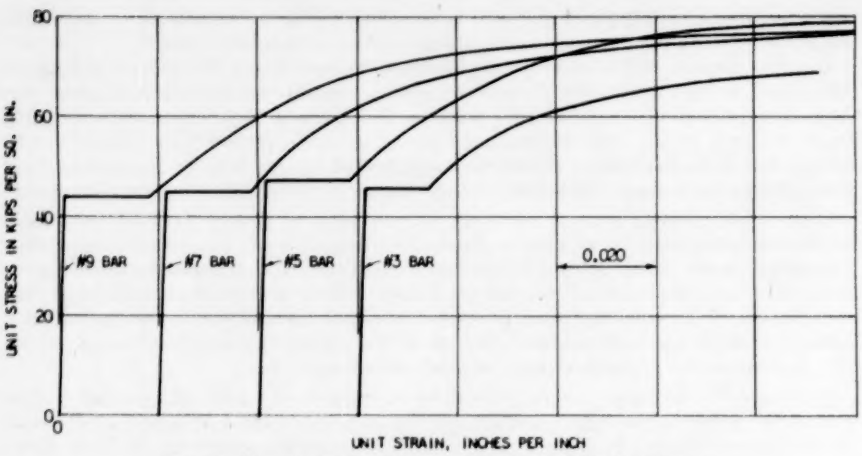


Fig. 3—Tensile stress-strain curves for reinforcing bars

both initial and stabilizing loads were recorded. That is, after reaching a strain value, the hydraulic control valves were manipulated so that the strain value was maintained, during which time the load would generally drop from the value first reached. A photograph of each beam was taken after test for Group 2S, but for Group 3S beams additional photographs were taken at first yield of the steel and at first evidence of concrete crushing, as well as at ultimate load.

A visual record of the development of collapse conditions for the F-groups was obtained for all but two specimens in motion pictures started prior to the plastic range of loading and continued until well past the ultimate load. This photographic record was then assembled into a film also containing a step-by-step description and pictures of the testing procedure.[5]

Test procedure for the F-groups was initiated by starting three dual-channel oscillographs simultaneously after all amplifiers had been warmed up, circuits balanced and calibrated, and pens zeroed. This was followed by the application of load, and starting of the electrically driven 16 mm motion picture camera. The loading rate was then increased until approximately maximum head speed had been attained in the plastic range. Table 3 gives a comparison of testing time with that specified for cylinders by ASTM. The test was stopped only after resistance to load had collapsed.

Total loads for Group 1F specimens were recorded on a single oscillograph channel from the two Baldwin SR-4 Type C load cells. For Groups 2F and 3F, the loads on each cell were recorded separately, in order to determine the distribution of load on the column stub after blocking the spherical head against rotation. Table 4 shows the position of the resultant of the loads on the two load cells at 10,000 pounds, concrete crushing, steel yield, at center line of column stub, and at maximum load. The distribution of load on the column stubs was

TABLE 2—PROPERTIES OF REINFORCING STEEL

Bar size	f_y psi*	ϵ_y in. per in.	ϵ_{sh} in. per in.	Ultimate strength, psi*	Ultimate unit strain, in. per in. × 100	Specimen group No.
#3	45,500	0.00152	0.0145	68,500	9.2	1, 2, & 3
#5	47,500	0.00158	0.0187	78,800	14.1	3
#7	45,000	0.00150	0.0190	76,600	14.5	2
#9	44,300	0.00148	0.0183	76,800	16.8	1

*Based on nominal area

satisfactory, insofar as maintaining an approximately uniform pressure was concerned. For the span of 9 ft, the greatest shift of the resultant would cause only a 3 percent change in the reactions as compared with a truly central load application.

TEST RESULTS

Principal test data

Principal test data are summarized in Tables 5 and 6. The load, moment, and deflection at steel yield, concrete crushing, and maximum are given in Table 5 for all beams, with the exception of values at steel yield for Groups 1F, 2F, and 3F. In the latter cases, the limited number of oscillograph channels prevented determination of steel strains at the critical section, if stub center line strains were to be recorded for extent of yield, so experimental values for yield at the critical section were not obtained for the F-series. Concrete and steel strains at concrete crushing and maximum load for the critical sections of the S-series are given in Table 6, with values of k_1k_3. The values were calculated on basis of equations presented in the following section on analytical studies.

Steel strain, in each case, is the average of all steel gage recordings at the critical sections. Concrete strain at crushing, in each case, is the average of all concrete gage recordings at the load when strain began to recede on any one or more gages. However, the concrete strain shown for maximum load is the maximum concrete strain from those gages still showing an increase in strain as the load approached P_{max}, without extrapolation, and generally at the load increment just preceding the maximum load.

Four arrangements of concrete gages were compared for obtaining strains at the critical section: ¾ in., 1¾ in., and 2¾ in. from the face of the column

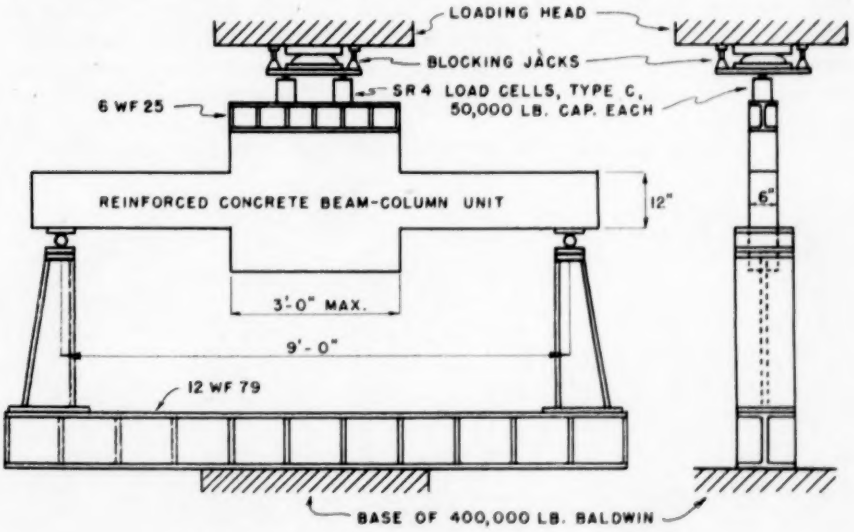


Fig. 4—Simulated beam-column connection test set-up

TABLE 3—TESTING TIME FOR F-SERIES

Specimen No.	Time to reach		Cylinder test time at 20 psi per sec
	Crushing, min	Maximum, min	
1- 6F	3.25	6.10	3.5 min
1-12F	3.34	4.55	
1-18F	2.33	2.85	
1-24F	2.86	5.27	
1-36F	1.64	4.07	
2- 6F	2.86	2.57*	3.5 min
2-12F	2.76	4.00	
2-18F	2.40	2.40*	
2-24F	3.18	2.97*	
2-36F	2.23	3.53	
3- 6F	2.09	3.31	2.8 min
3-12F	1.39	2.13	
3-18F	2.37	3.55	
3-24F	2.83	3.82	
3-36F	2.10	2.95	

*First maximums

stub on the center line of the top surface of the beam, and on the vertical faces at $\frac{3}{4}$ in. down from the top surface and $\frac{3}{4}$ in. from the face of the column in the direction of the reaction. No consistent or significant strain differences were shown for those beams carrying different arrangements for comparison. For example, note that in Fig. 6 for Group 1F, the gage at $\frac{3}{4}$ in. from the column face recorded crushing C second for 36F and 24F, first for 18F and 12F, and at the same time for 6F, in relation to the gage at $2\frac{3}{4}$ in. from the column face, C'.

Specimens 2-24F, 2-18F, and 6F developed an initial maximum followed by a slight load reduction at the beginning of the plastic region, but reached a second maximum at a somewhat lower load before collapse was evident. The initial maximums are shown on the load-deflection curves of Fig. 6 for Group 2F, but are not used for tabular data.

Behavior under load

If the tension steel ratio producing simultaneous development of the yield point in the tension reinforcement and crushing of the concrete on the compression face is defined as the *balanced* steel ratio, the general behavior of a beam in flexure should conform to one of the three cases illustrated in Fig. 5. The idealized relationships between resisting moment and unit rotation are shown at the top, but the lack of distinctly separated elastic and plastic properties in the concrete and the addition of compression reinforcement will modify the relationships to those given in (b). The relative variation of unit

TABLE 4—POSITION OF RESULTANT LOAD ON COLUMN

Group No.	Column width, in.	Distance of resultant from column centerline, in., at			
		10,000 lb.	Crushing	Steel yield*	Maximum load
2F	6	0.56	0.17	0.33	0.05
	12	0	0.24	0	0.18
	18	0	0.28	1.03	0.28
	24	0	0.58	0.28	0
	36	0.15	0.65	0.09	0.09
3F	6	0.23	0.32	0	0.24
	12	0.09	0.30	0.27	0.13
	18	0	0	0.15	0.51
	24	0	0.10	0.92	1.58
	36	0	0	No yield	1.15

*At center line of column

TABLE 5—COMPARISON OF EXPERIMENTAL LOADS, MOMENTS, AND DEFLECTIONS*

Specimen No.	Load, kips, at			Moment, in.-kips, at			Deflection, in., at		
	Yield	Crushing†	Maximum	Yield	Crushing	Maximum	Yield	Crushing	Maximum
1- 6S	34.5‡	28.0	34.5	880	715	880	0.78	0.40	0.78
1-12S	39.0‡	33.0	39.0	936	791	936	1.13	0.50	1.13
1-18S	42.0‡	36.0	42.0	945	810	945	1.02	0.52	1.00
1-24S	44.0‡	40.0	44.0	924	840	924	0.84	0.54	0.84
1-36S	50.0	44.0	50.5	900	792	910	0.63	0.42	0.92
1- 6F	—	26.4	36.6	—	674	934	—	0.25	0.85
1-12F	—	29.4	40.6	—	706	975	—	0.27	0.48
1-18F	—	38.7	43.7	—	870	980	—	0.35	0.52
1-24F	—	34.9	44.4	—	733	931	—	0.13	0.46
1-36F	—	37.6	54.0	—	676	972	—	0.09	0.36
2- 6S	28.2‡§	28.2	28.2	720	720	720	0.53	0.60	1.80
2-12S	29.8	30.6	30.7	715	735	736	0.45	0.78	1.01
2-18S	31.1	30.7**	31.6	700	690	711	0.45	0.76	1.47
2-24S	33.1	32.9**	33.8	695	691	709	0.50	0.71	1.27
2-36S	39.4	39.7	41.1	709	715	741	0.42	0.77	1.82
2- 6F	—	26.5	27.0††	—	676	688	—	0.52	0.85
2-12F	—	30.0	30.1	—	720	722	—	0.50	1.05
2-18F	—	31.6	30.0††	—	711	675	—	0.60	1.26
2-24F	—	32.0	32.5††	—	673	682	—	0.67	1.75
2-36F	—	34.8	40.0	—	626	720	—	0.36	1.64
3- 6S	10.7	12.5	14.3	274	320	366	0.58	1.96	4.96
3-12S	11.4	13.2	14.5	274	317	348	0.56	1.70	4.68
3-18S	12.0	13.5	15.5	270	304	349	0.51	1.11	3.51
3-24S	13.0	14.8	16.7	273	311	351	0.58	1.56	4.12
3-36S	15.4	17.7	19.9	277	318	358	0.56	1.65	3.62
3- 6F	—	11.2	13.8	—	286	352	—	1.13	3.78
3-12F	—	12.2	14.4	—	293	346	—	0.95	2.84
3-18F	—	13.8	15.8	—	310	355	—	1.23	3.88
3-24F	—	14.5	16.2	—	305	340	—	1.27	3.77
3-36F	—	17.0	19.7	—	306	354	—	1.29	3.70

*Total test time for S-series, 1 hr or more; for F-series, 2.1 min to 6.1 min.
†As indicated by gages nearest the critical section.
‡Steel yield developed at maximum load.
§Load remained constant at maximum at development of yield and crushing.
**Although $P_c < P_y$, crushing followed yield in time sequence.
††Second maximums.

TABLE 6—EXPERIMENTAL STRAINS, STRESSES, AND k_1k_3 VALUES, FOR $p' = 0.0037$ AND $f_y' = 45{,}500$ psi

Specimen No.	p	Steel strain,*† in. per in. × 10^6, at		Concrete strain,†‡ in. per in. × 10^6, at		Steel stress, f_s ksi	f_c', psi	k_1 k_3 § at	
		P_c	P_{max}	P_c	P_{max}			P_c	P_{max}
1- 6S	0.0500	1125	5740	2950	4390	33.8	4180	0.46	1.13
1-12S	0.0500	1180	5230	3050	4910	35.4	4180	0.53	0.96
1-18S	0.0500	1210	1640	2910	4590	36.3	4180	0.56	0.66
1-24S	0.0500	1230	3680	3740	5300	36.9	4180	0.53	0.83
1-36S	0.0500	1200	2010	2340	2960	36.0	4180	0.59	0.82
Average						35.7		0.53	0.88
2- 6S	0.0300	5480	10350	2040	2960	45.0	4170	1.05	1.28
2-12S	0.0300	11400	12800	4010	5360	45.0	4170	1.09	0.96
2-18S	0.0300	13350	16460	3450	5790	45.0	4170	1.39	1.09
2-24S	0.0300	9000	16520	2710	5530	45.0	4170	1.23	1.13
2-36S	0.0400	13150	—	4060	—	45.0	4170	1.20	—
Average						45.0		1.19	1.12
3- 6S	0.0102	22100	—	2970	—	51.2**	3300	0.92	—
3-12S	0.0102	22430	—	3290	—	51.6**	3300	0.86	—
3-18S	0.0102	19820	—	3170	—	49.0**	3300	0.74	—
3-24S	0.0102	20850	—	2720	—	50.0**	3300	0.91	—
3-36S	0.0102	24650	—	3100	—	53.5**	3300	1.04	—
Average						51.1		0.89	—

*Average strain of all gages at critical sections.
†Lack of values indicates all gages inoperative.
‡Values at P_c are the averages of concrete strains when strain began to recede on a gage. Value at P_{max} is the maximum concrete strain from those gages still recording an increase in strain as load approached P_{max}, without extrapolation.
§Computed from equation developed in section on analytical studies.
**Stress values in work hardening obtained from stress-strain curve.

rotations along a beam from a point of contraflexure to and including the junction with a column is developed from the moment diagram in (c), for a condition of maximum moment at the critical section.

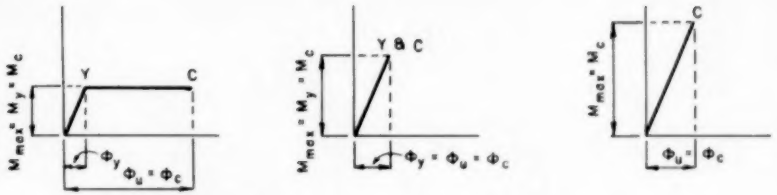
The degree of tension cracking and local bond failure in the region on both sides of the critical section determines the extent to which the maximum unit rotation tends to spread toward the center of the column and the point of contraflexure. For balanced reinforcement, with very little development of yield strain, and for over-reinforced sections, with crushing occurring before the yield point, there cannot be appreciable tension cracking or bond failure prior to maximum load. Therefore, at or above balanced steel ratios, the spread of maximum unit rotation will be slight, with the result that the plastic rotation between the beam and the column will be correspondingly small, and the deflection of the point of contraflexure relative to the column junction should be of the order of magnitude of elastic deflection. On the other hand, the steel strains for sections having less than balanced reinforcement will be beyond the initial yield point value, resulting in extensive tensile cracking and local bond failure, a tendency toward a large spread of maximum unit rotation, and a substantial deflection at maximum load. Typical load-deflection curves for the three conditions are sketched in (d).

From left to right, the three cases illustrated in Fig. 5 are typical of the behavior of Groups 3, 2, and 1 respectively, excepting that Group 2 was somewhat below balanced reinforcement. The shaded areas shown in the ϕ-diagrams of (c) are defined as the concentrated plastic rotation at maximum load for the test specimens reported herein.

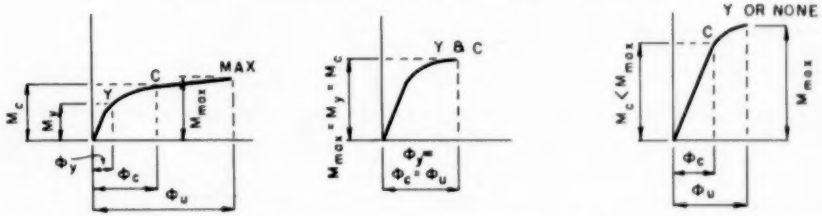
Load deflection curves

Fig. 6 and 7 provide the actual load-deflection curves obtained from test. Points C correspond to crushing of the outmost compression fibers as indicated by a decrease in the concrete strains. This decrease was interpreted to result from a release of strain on the gage during crushing. Points marked Y indicate the development of yield point strains in the steel, with N, S, and CL designating north or south column faces, and column center line, respectively.

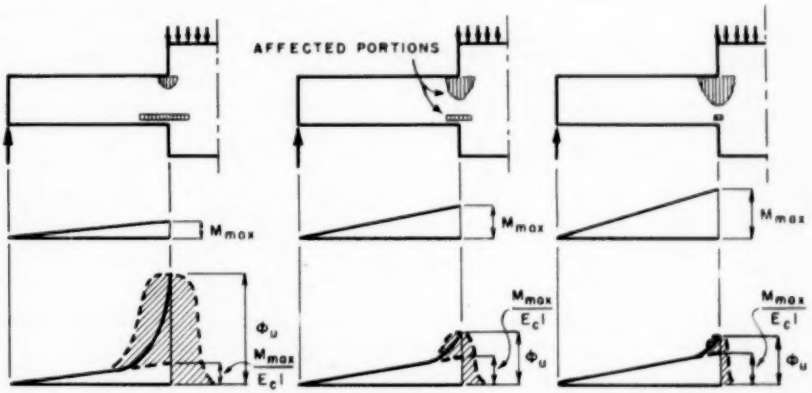
For Groups 1, 2, and 3 of both S- and F-series, the sequence of development of concrete crushing and steel yield is consistent with that described in Fig. 5, considering that Group 2 specimens have somewhat less than balanced reinforcement. Concrete crushing and maximum load occurred at markedly smaller deflections for the F-series than for those tested at a slower rate. Although steel yield at the column faces was not measured for the F-series, the result of the faster loading was undoubtedly to bring the development of yield at the column faces and concrete crushing closer together, and for Group 2F this resulted in virtually balanced reinforcement conditions. Specimens 24F, 18F, and 6F exhibited an initial maximum at what appears to be the point of development of steel yield at the column faces, followed by a slight drop and a second maximum at a slightly lower value. The second maximums



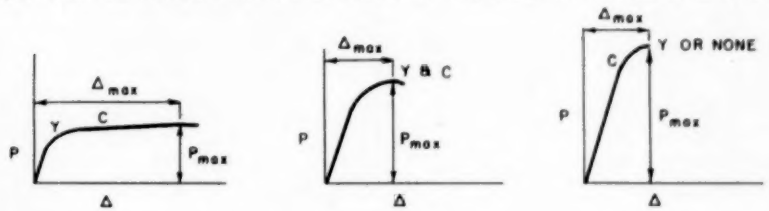
(a) IDEALIZED BEHAVIOR AT CRITICAL SECTION



(b) MODIFIED BEHAVIOR AT CRITICAL SECTION



(c) RELATIVE CONCENTRATION OF PLASTIC YIELDING AT MAXIMUM MOMENT



(d) TYPICAL LOAD-DEFLECTION CURVES

$q' < q'_{cr}$ $q' = q'_{cr}$ $q' > q'_{cr}$

Fig. 5—Behavior of test specimens

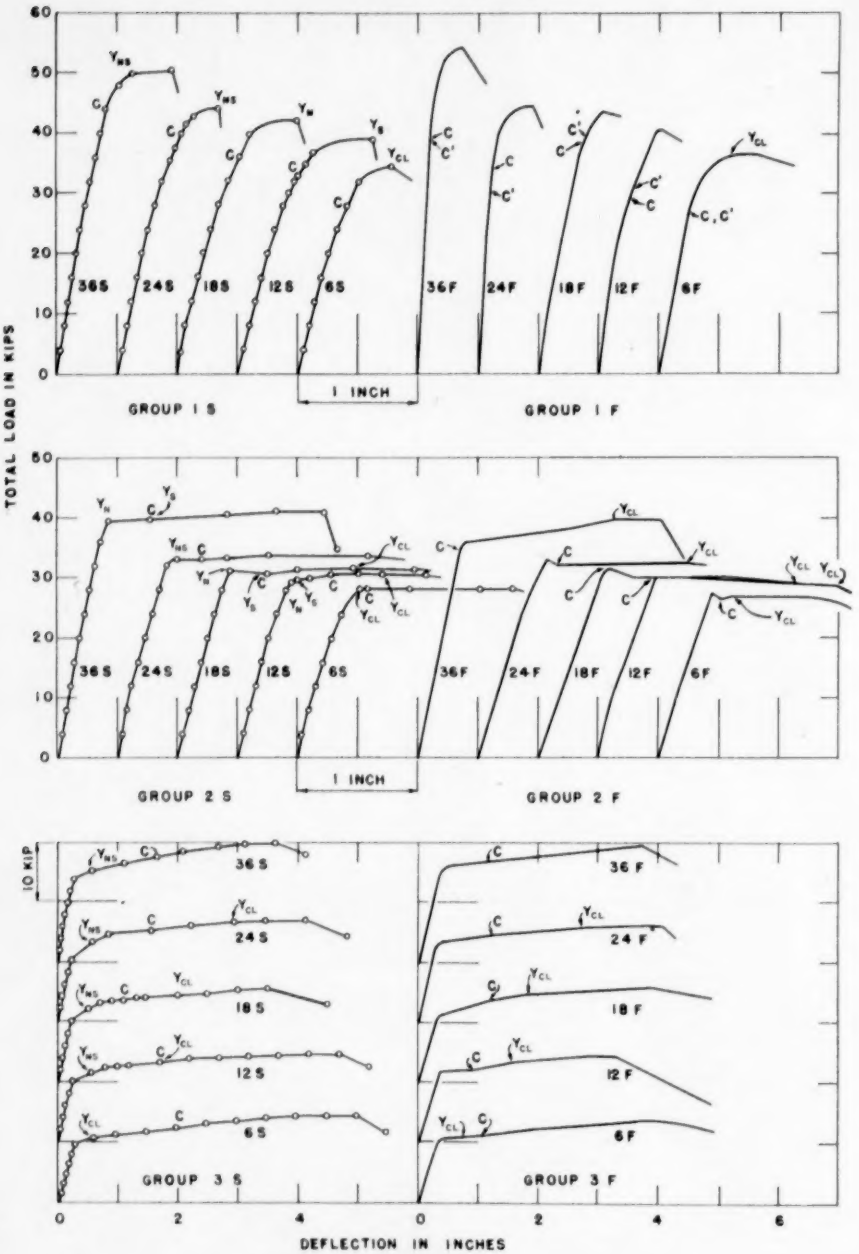


Fig. 6—Load-deflection curves

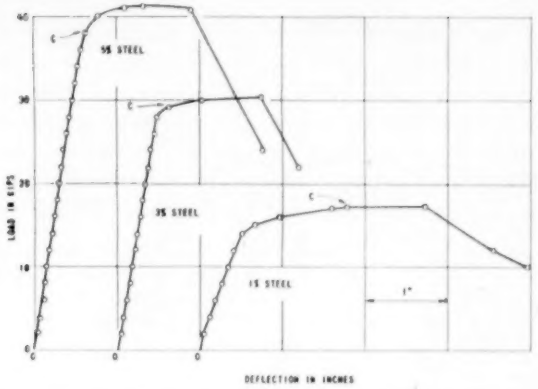


Fig. 7—Load-deflection curves for repaired beam-to-column connections

for these specimens were used for purposes of comparison with the other specimens.

Fig. 8 illustrates typical appearance of Group 3 beams at steel yield, concrete crushing, and at ultimate load. Major portion of the visible deflection developed between crushing and ultimate loads.

Load-strain curves

Typical load-strain curves are shown in Fig. 9 for specimen 1-24S. Steel strains of the same order of magnitude as those at the column faces were developed at the column center line during the early period of loading in all specimens. Yield at the center line is discussed in the following section on yield and crushing.

Yield and crushing

An objective of varying column stub width from 6 in. to 36 in. was to trace the spread of steel yield into the column at crushing of the concrete and at maximum load. Although results were more scattered and less conclusive than was hoped, certain trends are reasonably apparent. In Fig. 10, the ratios of center line yield deflection to the deflection at crushing or maximum load have been plotted against the corresponding values of half the column width for all specimens. Since the value of $w/2$ corresponding to a deflection ratio of unity for a specific steel ratio defines the maximum limit of yield spread, the values at unity for each steel ratio as obtained from the first four diagrams of Fig. 10, are replotted in the last diagram to show the relationship between maximum spread of yield into the column and steel ratio. The maximum spread of yield is somewhat reduced by fast loading, but the effect is not particularly significant at crushing, and is not conclusive at maximum load. The variation of maximum yield spread with steel ratio is a definite trend. Average values to the nearest in. for these tests are:

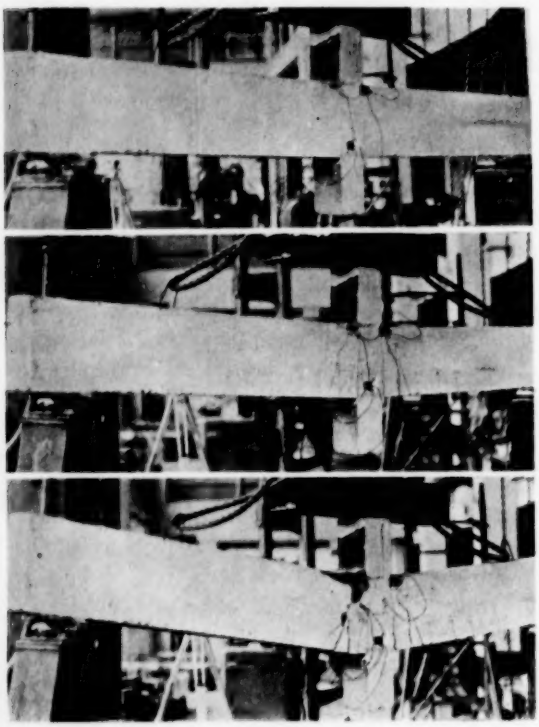
Group No.	Steel ratio	Maximum yield spread at	
		P_c	P_{max}
1	0.05	1 in.	3 in.
2	0.03	3 in.	6 in.
3	0.01	5 in.	15 in.

The length of fracture on the compression face should be a measure of the total extent of crushing strain in the concrete. Top surface measurements of the fractures at the critical sections were the same for the slowly and rapidly tested specimens, within the accuracy of measurement possible, but varied with the amount of longitudinal reinforcement. Difficulty in estimating the portion actually affected by compression crushing was caused by spalling on the sides and top, but the resulting average values to the nearest in. were

Group No.	Steel ratio	Crushed length at critical section
1	0.05	8.0
2	0.03	6.0
3	0.01	5.0

Fig. 11 indicates appearance of typical fractures at collapse.

It must be noted that these specimens were heavily reinforced against diagonal tension failure, with the result that the concrete was constrained more than if the transverse steel had been the optimum for prevention of diagonal tension failure. The spread of plastic strains on the compression face and into the column should be considerably greater than the crushed length as given above.



(a) At start of steel yield at center line of column

(b) At crushing of the concrete, as indicated by strain release on SR-4 gages

(c) At collapse

Fig. 8—Specimen 3-6S during test

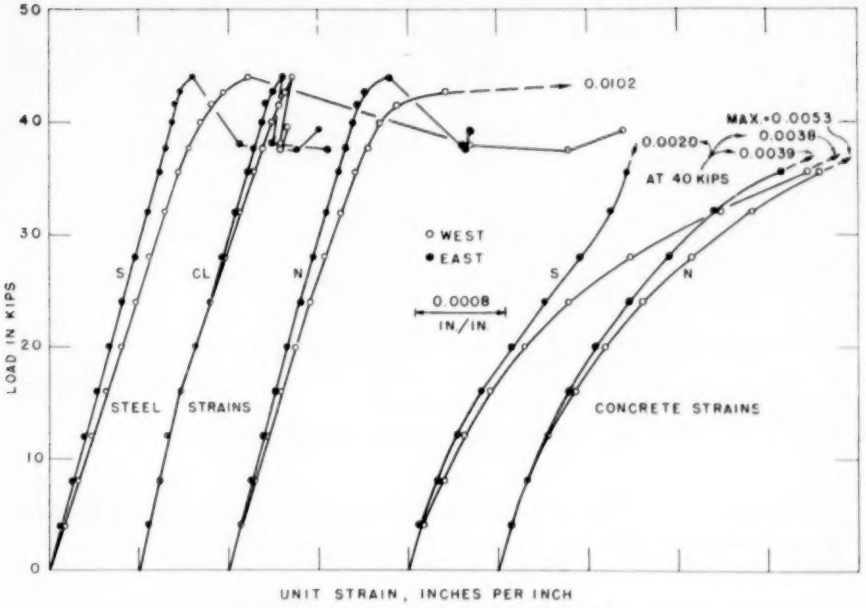


Fig. 9—Load-strain curves for 1-24S

Repaired sections

Three specimens, originally tested under rapid loading, were selected for repairing of the failed sections. The 6-in. column stub specimen from each steel ratio group was taken because of the relatively greater damage. Each specimen was inverted, then placed in the testing machine and straightened. All damaged concrete was removed, taking care to chisel out approximately square for all surfaces to be in contact with fresh concrete. The open portion was then formed and cast with fresh concrete made in a laboratory mixer. The average strength of three 6 x 12-in. cylinders at time of test was 4860 psi.

The load-deflection curves, obtained with slow loading, are shown in Fig. 7. Maximum loads are greater for the repaired sections, but the effective depths were 5 to 10 percent greater than originally, because of the kink developed from the first test. Also, the tension reinforcement in 3-6F had been stressed into the work-hardening range in the original test, as is evident in the higher position of yield on the retest curve.

ANALYTICAL STUDIES

Theory

Expressions are already available[1,6,7] with which values of moment may be computed for comparison with these experimental findings. Also, the relationship between unit rotations developed in flexure and the deflection at any point on a beam has been presented.[8,9,10,11] Only expressions directly

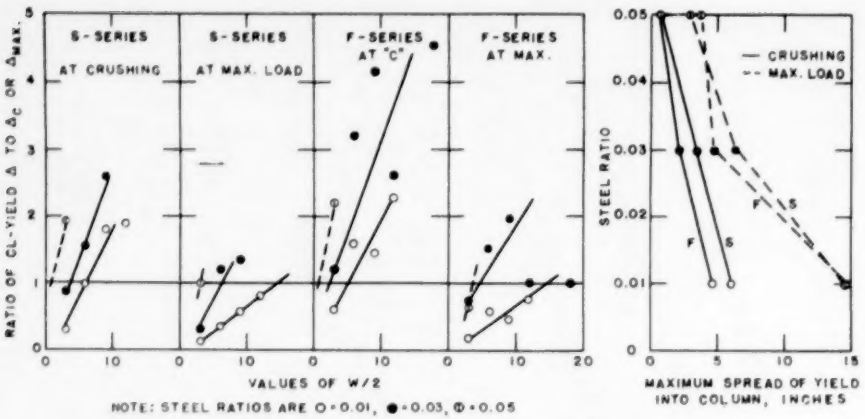

S-SERIES AT CRUSHING
S-SERIES AT MAX. LOAD
F-SERIES AT "C"
F-SERIES AT MAX.
RATIO OF CL-YIELD Δ TO Δ_c OR $\Delta_{MAX.}$
VALUES OF W/2
NOTE: STEEL RATIOS ARE ○ = 0.01, ● = 0.03, Φ = 0.05
CRUSHING
MAX. LOAD
STEEL RATIO
MAXIMUM SPREAD OF YIELD INTO COLUMN, INCHES

Fig. 10—Maximum spread of steel yield

useful to analytical study of these tests are given below. In general, developments are not provided unless helpful to an understanding of an equation or its use.

Elastic equations—Customary assumptions of linear distribution of strain, proportionality of stress to strain, and no tension in the concrete, are the basis for the following equations. Refer to Fig. 12(a). The equations are developed at the limit $f_s = f_y$.

From equality of total forces of tension and compression:

$$f_c = \frac{p\,f_y - p'f_s'}{\frac{k}{2}} \qquad (1)$$

From strain distribution:

$$\frac{\epsilon'}{\epsilon_c} = \frac{k' + k - 1}{k}$$

$$\frac{f_s' E_c}{f_c E_s} = \frac{k' + k - 1}{k}$$

$$f_s' = \frac{k' + k - 1}{k}\, n\, f_c \qquad (2)$$

Substituting Eq. (2) in Eq. (1):

$$f_c = \frac{p\,f_y}{\frac{k}{2} + \frac{n\,p'}{k}(k' + k - 1)} \qquad (3)$$

From the transformed section:

$$k = \sqrt{[(n-1)p' + np]^2 + 2[(n-1)(1-k')p' + np]} - [(n-1)p' + np] \quad (4)$$

$$I = \left[\frac{k^3}{3} + p'(n-1)(k' + k - 1)^2 + np(1-k)^2\right] bd^3 \quad (5)$$

Resisting moment at yield of tension steel, with concrete in elastic range is:

$$M_y = \frac{f_c}{2} kbd^2\left(1 - \frac{k}{3}\right) + A_s' f_s' k' d \quad (6)$$

Curvature or unit rotation, for elastic strains is:

$$\phi = \frac{M}{E_c I} \quad (7)$$

Modulus of elasticity of concrete[12] is:

$$E_c = 1{,}800{,}000 + 460 f_c' \quad (8)$$

Plastic equations—The assumptions of linear distribution of strain, plastic compression stress-block, and no tension in the concrete, are the basis for plastic equations given on the following page. Refer to Fig. 12(b).

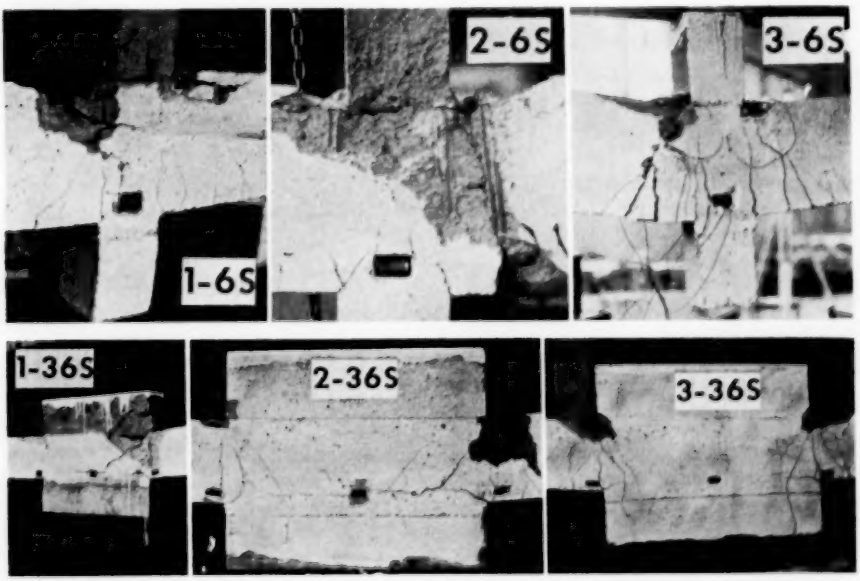
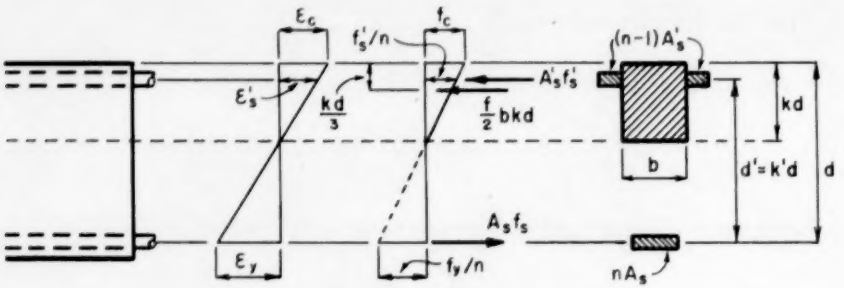
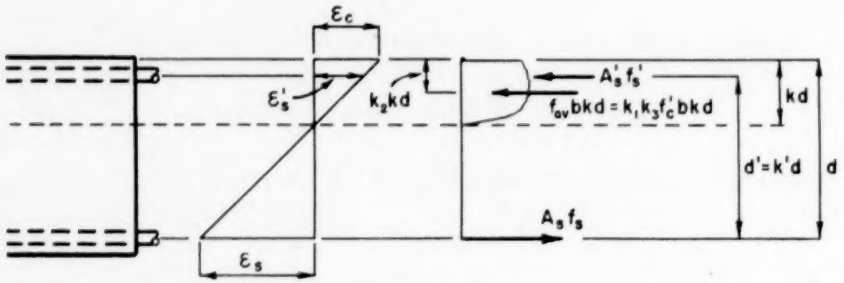


Fig. 11—Typical fractures at collapse



(a) ELASTIC



(b) PLASTIC

Fig. 12—Distribution of stress and strain for analytical studies

From the equality of total forces of tension and compression:

$$k = \frac{p\,f_s - p'\,f_s'}{f_{av}} \qquad (9)$$

From strain distribution:

$$k = \frac{\epsilon_c}{\epsilon_c + \epsilon_s} \qquad (10)$$

Combining Eq. (9) and (10), and substituting $k_1\,k_3\,f_c'$* for f_{av}:

$$\frac{p\,f_s - p'\,f_s'}{f_c'} = \frac{\epsilon_c}{\epsilon_c + \epsilon_s}\,k_1\,k_3 \qquad (11)$$

If concrete crushing precedes yield in tension steel:

$$\frac{p\,f_s - p'\,f_y'}{f_c'} = \frac{\epsilon_u}{\epsilon_u + \epsilon_s}\,k_1\,k_3 \qquad (12)$$

*k_1, k_3 as used in this study is the ratio of average concrete stress to cylinder strength, whether or not the concrete is fully developed in compression.

If concrete crushing and yield in tension steel occur simultaneously:

$$\frac{p\,f_y - p'\,f_y'}{f_c'} = \frac{\epsilon_u}{\epsilon_u + \epsilon_s}\,k_1\,k_2 \qquad (13)$$

The right term of Eq. (13) defines the strain relationship for balanced reinforcement. If the left term, calculated from actual beam and material properties, is greater than the right term, failure should occur by crushing of the concrete prior to yield in the tension steel. The left term is the reinforcing index, q', for a section reinforced in tension and compression, and the right-hand term defines the limit of q' for balanced reinforcement, as follows:

$$q' = \frac{p\,f_y - p'\,f_y'}{f_c'} \qquad (14)$$

$$q'_{cr} = \frac{\epsilon_u}{\epsilon_u + \epsilon_y}\,k_1\,k_2 \qquad (15)$$

The limiting value for work hardening is:

$$q'_{wh} = \frac{\epsilon_u}{\epsilon_u + \epsilon_{wh}}\,k_1\,k_2 \qquad (16)$$

General plastic moment equation is:

$$M = k_1\,k_2\,f_c'\,k\,b\,d^2\,(1 - k_2\,k) + A_s'\,f_s'\,d' \qquad (17)$$

At concrete crushing, and for the amount and position of the compression reinforcement of these tests, the expression for k from Eq. (9) becomes

$$k = \frac{p\,f_s - p'\,f_y'}{k_1\,k_2\,f_c'} \qquad (18)$$

Substituting Eq. (18) in Eq. (17), with $f_s' = f_y'$:

$$M_c = \left\{(pf_s - p'\,f_y')\left[1 - \frac{k_2}{k_1 k_2}\left(\frac{pf_s - p'\,f_y'}{f_c'}\right)\right] + p'\,f_y'\,k'\right\} bd^2 \qquad (19)$$

If the section is capable of sustaining additional moment after crushing of the outer concrete fibers, Eq. (19) may be used for computing M_{max} if f_s, k_1k_3, and k_2 are the values at the maximum moment instead of at crushing. Values of f_s and k_1k_2 at C and P_{max} are computed from the test data in Table 6, for use with Eq. (19).

Insofar as k_2 is concerned, experimental studies by others[12] have shown values ranging from approximately 1/3 at low loads to over ½ at failure. The value of 0.5 has been selected for this study, at crushing and maximum loads.

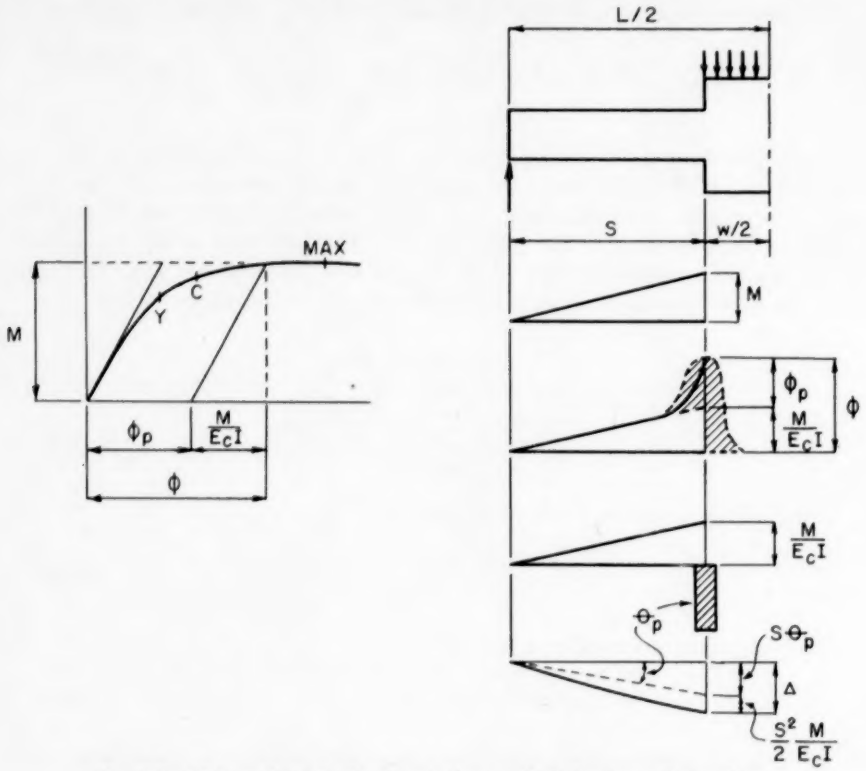


Fig. 13—Unit and total plastic rotation, and deflection relationships

The concentrated plastic rotation Θ_p for the type of specimen and loading used in these tests may be developed from Fig. 13 as below:

$$\Delta = \frac{2}{3} S \frac{S}{2} \frac{M}{E_c I} + S \Theta_p$$

$$\Theta_p = \frac{\Delta}{S} - \frac{M}{E_c I} \frac{S}{3} \quad \ldots\ldots\ldots\

TABLE 7—EXPERIMENTAL AND ANALYTICAL YIELD MOMENTS AND CONCENTRATED PLASTIC ROTATION AT YIELD

Specimen No.	Yield moment, in.-kips		Experimental M_y / Analytical M_y	Δ, in., measured at yield	θ_p, radians, at yield
	Experimental	Analytical			
1- 6S	880	1050	0.84	0.78	0.0106
1-12S	936	998	0.94	1.13	0.0187
1-18S	945	856	1.10	1.00	0.0181
1-24S	924	950	0.97	0.84	0.0159
1-36S	900	945	0.95	0.63	0.0140
Average Group 1-S			0.96		0.0155
Range			0.84–1.10		0.0106–0.0187
2- 6S	720	698	1.03	0.53	0.0052
2-12S	715	705	1.01	0.45	0.0045
2-18S	700	722	0.97	0.45	0.0055
2-24S	695	716	0.97	0.50	0.0077
2-36S	709	710	1.00	0.42	0.0080
Average Group 2-S			1.00		0.0062
Range			0.97–1.03		0.0045–0.0080
3- 6S	274	260	1.05	0.58	0.0071
3-12S	274	260	1.05	0.56	0.0076
3-18S	270	260	1.04	0.51	0.0076
3-24S	273	260	1.05	0.58	0.0103
3-36S	277	260	1.07	0.56	0.0125
Average Group 3-S			1.05		0.0090
Range			1.04–1.07		0.0071–0.0125
Over-all average S-series			1.00		
Range			0.84–1.07		

NOTE—Yield moments and deflections were not obtained experimentally for the F-series. Eq. (19) was used for analytical moments for Groups 1 and 2. Eq. (6) was used for analytical moments for Group 3.

Moments at yield were computed for the S-series only, because experimental yield moments were not obtained for the F-series. For Group 1S, the moments at yield are the same as the maximum moments computed from Eq. (19) with k_1k_3 and f_s as listed for P_{max}. Yield and crushing developed at virtually the same loads for Group 2S, requiring the use of k_1k_3 and f_s at P_c in Eq. (19), and therefore resulting in the same values as crushing moment. Yield moments for Group 3S were computed from Eq. (6).

These analytical moments for each specimen are listed in Tables 7, 8, and 9 together with experimentally determined moments and the ratios of experimental to analytical values. Considering both S- and F-series, the ratios for the individual specimens range from 0.84 to 1.24. The over-all averages for the S-series are 1.00, 1.07, and 1.02, for yield, crushing, and maximum moment, respectively; and for the F-series the over-all averages for crushing and maximum moments are 1.02 and 1.01.

TABLE 8—EXPERIMENTAL AND ANALYTICAL CRUSHING MOMENTS AND CONCENTRATED PLASTIC ROTATION AT CRUSHING

Specimen No.	Crushing moment, in.-kips		Experimental M_c / Analytical M_c	Δ, in., measured at crushing	θ_p, radians, at crushing
	Experimental	Analytical			
1- 6S	715	639	1.12	0.40	0.0040
1-12S	791	699	1.13	0.50	0.0064
1-18S	810	726	1.11	0.52	0.0077
1-24S	840	711	1.18	0.54	0.0091
1-36S	792	741	1.07	0.42	0.0086
Average Group 1-S			1.12		0.0072
Range			1.07–1.18		0.0040–0.0091
1- 6F	674	701	0.96	0.25	0.0013
1-12F	706	701	1.01	0.27	0.0020
1-18F	870	701	1.24	0.35	0.0036
1-24F	733	701	1.04	0.13	−0.0016
1-36F	676	701	0.97	0.09	−0.0009
Average Group 1-F			1.04		0.0009
Range			0.96–1.24		−0.0009–0.0036
2- 6S	720	698	1.03	0.60	0.0066
2-12S	735	705	1.04	0.78	0.0112
2-18S	690	722	0.96	0.76	0.0125
2-24S	691	716	0.96	0.71	0.0128
2-36S	715	710	1.01	0.77	0.0177
Average Group 2-S			1.00		0.0122
Range			0.96–1.04		0.0066–0.0177
2- 6F	676	710	0.95	0.52	0.0053
2-12F	720	710	1.01	0.50	0.0055
2-18F	711	710	1.00	0.60	0.0088
2-24F	673	710	0.95	0.67	0.0120
2-36F	626	710	0.88	0.36	0.0068
Average Group 2-F			0.96		0.0077
Range			0.88–1.01		0.0053–0.0120
3- 6S	320	290	1.10	1.96	0.0334
3-12S	317	290	1.09	1.70	0.0307
3-18S	304	280	1.09	1.11	0.0205
3-24S	311	282	1.10	1.56	0.0331
3-36S	318	303	1.05	1.65	0.0424
Average Group 3-S			1.09		0.0320
Range			1.05–1.10		0.0205–0.0424
3- 6F	286	287	1.00	1.13	0.0177
3-12F	293	287	1.02	0.95	0.0154
3-18F	310	287	1.08	1.23	0.0230
3-24F	305	287	1.06	1.27	0.0264
3-36F	306	287	1.07	1.20	0.0300
Average Group 3-F			1.05		0.0225
Range			1.00–1.08		0.0177–0.0300
Over-all average S-series			1.07		
Range			0.96–1.18		
Over-all average F-series			1.02		
Range			0.88–1.24		

TABLE 9—EXPERIMENTAL AND ANALYTICAL MAXIMUM MOMENTS AND CONCENTRATED PLASTIC ROTATION AT MAXIMUM

Specimen No.	Maximum moment, in.-kips		Experimental M_{max} / Analytical M_{max}	Δ, in. measured at maximum	Θ_p, radians, at maximum
	Experimental	Analytical			
1- 6S	880	1050	0.84	0.78	0.0106
1-12S	936	998	0.94	1.13	0.0187
1-18S	945	856	1.10	1.02	0.0181
1-24S	924	950	0.97	0.84	0.0159
1-36S	910	945	0.96	0.92	0.0220
Average Group 1-S			0.96		0.0171
Range			0.84–1.10		0.0106–0.0220
1- 6F	934	973	0.96	0.85	0.0116
1-12F	975	973	1.00	0.48	0.0050
1-18F	980	973	1.01	0.52	0.0069
1-24F	931	973	0.96	0.46	0.0068
1-36F	972	973	1.00	0.36	0.0063
Average Group 1-F			0.99		0.0073
Range			0.96–1.01		0.0050–0.0116
2- 6S	720	716	1.01	1.80	0.0300
2-12S	736	698	1.05	1.01	0.0160
2-18S	711	705	1.01	1.47	0.0280
2-24S	709	705	1.01	1.27	0.0260
2-36S	741	705	1.05	1.82	0.0460
Average Group 2-S			1.03		0.0292
Range			1.01–1.05		0.0160–0.0460
2- 6F	688*	705	1.01	0.85	0.0117
2-12F	722*	705	1.02	1.05	0.0170
2-18F	675*	705	1.01	1.26	0.0237
2-24F	682*	705	0.98	1.75	0.0376
2-36F	720	705	1.02	1.64	0.0419
Average Group 2-F			1.01		0.0264
Range			0.98–1.02		0.0117–0.0419
3- 6S	366	335†	1.09	4.96	0.0915
3-12S	348	335†	1.04	4.68	0.0923
3-18S	349	335†	1.04	3.51	0.0732
3-24S	351	335†	1.05	4.12	0.0937
3-36S	358	335†	1.07	3.62	0.0963
Average Group 3-S			1.06		0.0894
Range			1.04–1.09		0.0732–0.0963
3- 6F	352	335†	1.05	3.78	0.0686
3-12F	346	335†	1.03	2.84	0.0531
3-18F	355	335†	1.06	3.88	0.0814
3-24F	340	335†	1.02	3.77	0.0855
3-36F	354	335†	1.06	3.70	0.0989
Average Group 3-F			1.04		0.0775
Range			1.02–1.06		0.0531–0.0989
Over-all average S-series			1.02		
Range			0.84–1.10		
Over-all average F-series			1.01		
Range			0.96–1.06		

*Second maximums

†Gages inoperative at P_{max} for Group 3. Analytical M_{max} based upon $kk_3 = 0.9$, $\epsilon_u = 0.0059$ (maximum $\epsilon_t = 0.00587$), resulting in $f_s = 60{,}000$ psi in work-hardening range on stress-strain curve.

Concentrated plastic rotations

Values for the concentrated plastic rotation at yield, crushing, and maximum loads for each specimen were computed from Eq. (20) and entered opposite the measured deflections in Tables 7, 8, and 9. These concentrated plastic rotations for the individual specimens of the S-series and the averages for Groups 1F, 2F, and 3F are plotted against steel ratios in Fig. 14. It is evident that the plastic rotation at steel yield or at concrete crushing is not large, and is not a major portion of that available at maximum load. The plastic rotations occasionally show a tendency to increase somewhat as the column width is increased, which is probably due to the increased opportunity for spread of strain into the column.

The fast loading rate, equal to that specified for cylinders, somewhat reduces the plastic rotation. It is to be expected that such a stress rate would not permit an appreciable flow or spread of strain at a critical section having a reduction in moment on each side, and it should be noted that the plastic rotation at crushing for the steel ratio of 0.05 is practically negligible at the fast loading rate.

Intersection of the curves for plastic rotation at steel yield and concrete crushing, if plotted from the same coordinate axes for a single value of f_c' would define the balanced steel ratio for which yield and crushing develop simultaneously under the conditions of these tests. With the cylinder strength of about 4200 psi for the steel ratios of 0.03 and 0.05, the curves of Fig. 14 intersect at $p = 0.038$ for slow loading. If it is assumed that yield develops for fast loading at the same deflection as for slow loading, the value for balanced p would be close to 0.03, which is consistent with the load-deflection

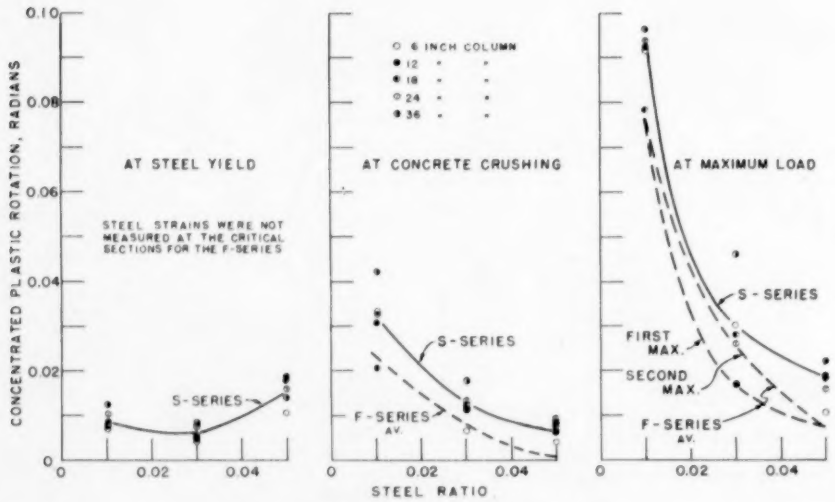


Fig. 14—Concentrated plastic rotations

curves for Group 2F on which the points for concrete crushing appear to be very close to that for yield.

SUMMARY

Thirty simulated beam-to-column connections of reinforced concrete were tested to determine the amount of plastic rotation that could be considered to be concentrated at the junction of the beam with the column, and also to evaluate the distance into a column to which yield strains would spread in the tension reinforcement. Half of the connections were tested at a stress rate on the compression face equivalent to that specified for concrete cylinders, in order to produce flow conditions approximately the same as that in standard cylinder tests. Also, three of the connections carried to failure under the rapid loading were repaired and retested for comparing with the results from the original tests.

The results from this entire investigation may be summarized as follows:

1. The sequence of steel yield, concrete crushing, and maximum load is typical of beams having less than balanced reinforcement, whereas concrete crushing followed by a maximum load with or without steel yield is the typical order for over-reinforced sections.
2. Values for maximum spread of steel yield into a column were 1, 3, and 5 in. for $p = 0.05$, 0.03, and 0.01, respectively, at concrete crushing; at maximum moment, the values were 3, 6, and 15 in. for $p = 0.05$, 0.03, and 0.01, respectively.
3. Lengths of fracture on the compression faces were approximately 8, 6, and 5 in. for $p = 0.05$, 0.03, and 0.01, respectively, but these values may have been influenced by the spacing of the diagonal tension reinforcement. These values are not considered a measure of spread of plastic strains on the compression face.
4. Moments computed from the analytical expressions presented herein are in good agreement with the moment determined experimentally.
5. A loading rate comparable to that for concrete cylinders reduces the amount of plastic rotation to some extent, in relation to a loading rate that permits manual reading of load, strain, and deflection.
6. The balanced steel ratio for the f_c' of approximately 4200 psi for Groups 1S and 2S was experimentally determined to be 0.038, and for Groups 1F and 2F the balanced ratio was about 0.030.
7. Plastic rotations at steel yield and at concrete crushing were not large, and were not a major portion of that available at maximum load. The maximum average value of plastic rotation for slow loading was 0.089 radians for $p = 0.01$, but reduced to 0.029 radians for $p = 0.03$, and 0.017 radians for $p = 0.05$; corresponding maximum average values for rapid loading were 0.078, 0.016, and 0.007 radians, all at maximum load.
8. Results from tests on repaired sections were not significantly different from the original test results, although maximum loads were somewhat higher apparently due to a lowering of the center of gravity of the steel at the repaired section. The effect of work hardening in raising the load at steel yield was also evident.

ACKNOWLEDGMENTS

This investigation was carried out in the testing laboratory of the department of engineering mechanics of the University of Nebraska, through the courtesy of Prof. E. J. Marmo, chairman of the department.

The project was conducted under a grant from the National Science Foundation for a general study of reinforced concrete connections in the plastic

range. Certain necessary pieces of laboratory equipment of a permanent nature were purchased from funds of the University Research Council and the George P. Abel Memorial Fund of the University of Nebraska Foundation.

Acknowledgment is given to Prof. G. M. Smith and Prof. Lyle E. Young, of the department of engineering mechanics, and to Marvin Garber, graduate student in electrical engineering, for assistance during testing of the specimens.

REFERENCES

1. Gaston, J. B., Siess, C. P., and Newmark, N. M., "An Investigation of the Load-Deformation Characteristics of Reinforced Concrete Beams up to the Point of Failure," *Civil Engineering Studies—Structural Research Series*, No. 40, University of Illinois, 1952.

2. McCollister, H. M., Siess, C. P., and Newmark, N. M., "Load-Deformation Characteristics of Simulated Beam-Column Connections in Reinforced Concrete," *Civil Engineering Studies—Structural Research Series*, No. 76, University of Illinois, 1954.

3. Glanville, W. H., and Thomas, F. G., "Studies in Reinforced Concrete—Section V," *Technical Paper*, No. 22, Department of Scientific and Industrial Research, England, 1939.

4. Garber, Marvin, "Electro-Mechanical Transducers for Structural Testing," unpublished thesis in partial fulfillment of the requirements for the master's degree, University of Nebraska, 1956.

5. "Load, Strain, and Deflection of Beam-to-Column Connections," University of Nebraska Film, June 1956.

6. Hognestad, E., "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," *Bulletin* No. 399, University of Illinois Engineering Experiment Station, 1951.

7. Whitney, C. S., "Plastic Theory of Reinforced Concrete Design," *Transactions*, ASCE, V. 107, 1942, p. 251.

8. Ernst, G. C., "Ultimate Slopes and Deflections," *Transactions*, ASCE, V. 121, 1956, p. 605.

9. Lee, L. H. N., "Inelastic Behavior of Reinforced Concrete Members Subjected to Short-Time Static Loads," *Transactions*, ASCE, V. 120, 1955, p. 181.

10. Sawyer, H. A., Jr., "The Behavior of Under-Reinforced Concrete Beams Under Long Term Loads," *Bulletin* No. 12, University of Connecticut Engineering Experiment Station, 1956.

11. Chan, W. L., "The Ultimate Strength and Deformation of Plastic Hinges in Reinforced Concrete Frameworks," *Magazine of Concrete Research* (London), V. 7, No. 21, Nov. 1955.

12. Hognestad, Eivind, Hanson, N. W., and McHenry, Douglas, "Concrete Stress Distribution in Ultimate Strength Design," ACI JOURNAL, Dec. 1955, *Proc.* V. 52, pp. 455-480.

Discussion of this paper should reach ACI headquarters in triplicate by Sept. 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

Title No. 53-64

Mass Concrete Control in Detroit Dam*

By ROY R. CLARK†

SYNOPSIS

Planning temperature control procedures for the mass concrete of Detroit Dam in western Oregon is explained in detail. Construction methods, including cooling practices, are given, along with cement types and proportioning of the concrete. Temperature history of the mass concrete supplements the analysis of temperature strains in each of four zones of the dam.

Uniformly stable conditions were established at the base of the dam, and inspection 3 years after completion disclosed no defects on the outside surface, and only a few hairline cracks on exposed interior surfaces. Total cost of cooling measures was $0.63 per cu yd, about 4 percent of the total cost of mass concrete.

INTRODUCTION

Detroit Dam, a multiple-purpose project, is located on the North Santiam River, 40 airline miles from Salem, the state capitol of Oregon. The reservoir surface, 1574.0 ft above mean sea level, is in the foothills of the Cascade Mountains, and the dam is subject to sharp temperature changes. This structure was designed and its construction supervised by the Corps of Engineers, United States Army. This paper studies one phase of the design, namely mass concrete control.

The dam is an overflow type with six radial gates which will discharge 176,000 cfs when fully open. Four high head outlet conduits in the dam will discharge in excess of 20,000 cfs depending on the elevation of water in the reservoir. A test conduit 8 ft in diameter was constructed through the dam at an elevation 234 ft below the maximum pool level. Two 15-ft diameter penstocks pass through the dam 170 ft below the maximum pool level.

Total concrete volume of the dam was 1,350,500 cu yd, all of which was subjected to the type of control to be described herein. Fig. 1 shows arrangement of concrete production facilities. Dam height from stilling basin foundation to roadway deck is 413 ft. Deep, irregular foundation excavations extended below this level approximately 60 ft in some places. Conventional foundation grouting and drainage were carried out. The base width of the dam at the highest section is 334 ft. Due to this wide base and the desire to eliminate contraction joints parallel with the long axis, the question of temperature control became of first importance. The design could not be com-

*Received by the Institute Oct. 8, 1956. Title No. 53-64 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53. Separate prints are available at 50 cents each. **Discussion** (copies in **triplicate**) should reach the Institute not later than Sept. 1, 1957. Address P. O. Box 4754, Redford Station, Detroit 19, Mich.

†Member American Concrete Institute, Civil Engineer, formerly Head of Design for Dams, Portland District, U. S. Army Corps of Engineers, Portland, Ore.

Fig. 1—Left abutment of Detroit Dam under construction. Note location of aggregate source, concrete plant, and cooling system

pleted until temperature studies progressed far enough to make possible a decision whether or not to eliminate vertical construction joints parallel with the axis. The basic design principles were determined in 1942-1943, but construction was postponed until March 1949 and completed in June 1953.

Foundation rock at the dam site is andesite of a single massive lithologic unit. Numerous fractures do not detract from the structural soundness of the rock but do form planes along which seepage may occur.

Contraction joints in the dam were not grouted or closed with waterstops at the downstream face. Also where the monoliths rest on bedrock the contraction joints were not sealed, permitting drainage at any place where water may exist in the foundation rock.

DESIGN CONSIDERATIONS

General

Newly placed concrete undergoes a rise in temperature due to the exothermic reaction of hydration of cement. In a concrete mass of nominal thickness, heat thus developed is readily dissipated and the thermal expansion of the concrete remains within acceptable limits. As the thickness of the mass increases, however, the additional concrete acts as an insulating barrier, decreasing the rate of heat dissipation. This correspondingly increases the temperature of the interior of the mass and causes thermal expansion greater than that near the exterior. Also, severe thermal shock may occur when the surface cools quickly while the interior is still heating. Such temperature changes may fracture the mass, destroy its monolithic character, radically alter the design stress distribution, and reduce the life of the structure.

To achieve best results, both at the interior and the surface of a structure, measures can be adopted to prevent excessive surface tension and to limit the temperature rise. Mass concrete control can therefore be defined as the utilization of those factors which keep surface and internal tensile stresses within the tensile strength of the concrete; if properly carried out it will provide a factor of safety commensurate with other elements of the structure design. The term "mass concrete" is here defined as portland cement concrete involved in a structure of such massive dimensions that temperature rise, if uncontrolled and due largely to the hydration of the cement, would result in objectionable thermal expansion of the concrete.

A gravity dam is a rigid structure which deflects very little. The volume change evidenced by measured strain is extremely small. These massive structures have frequently cracked, often in an unpredictable manner. Mass concrete containing cracks in any position causing ultimate weakness under stress must be considered defective. Tensile stresses which cause cracking are directly related to the thermal coefficient of expansion and to the modulus of elasticity of the concrete, and inversely related to the creep or plastic yielding of the concrete. Among existing dams of varying age in the United States, it has been noted that those in which the concrete has a low modulus of elasticity are now in better condition than those with higher modulus.

In the design of the dam, why should a stress analysis be considered complete without including a realistic coverage of temperature tensile stresses to determine whether or not structural ruptures may develop at the base in the early life of the structure and later extend upward through the mass to become problems of safety and permanence? The writer believes that such a weakness in many dams can be prevented by careful advance planning and adoption of adequate control measures without undue cost or delay in construction. Some of the steps to be taken are related to construction but in reality are design principles.

Improved methods of construction have encouraged and made possible more massive structures such as Detroit Dam. The tendency, however, is to construct too fast and to be opposed to special methods to improve quality which are not usually understood or accepted. On the other hand, modern construction techniques can aid in proper concrete control methods. More accurate control of batching and placing is possible, vibration and use of air entrainment encourages lean mixes, producing homogeneous concrete that lends itself to more accurate control measures.

Temperature control

The control of temperature rise in mass concrete is sometimes referred to as an effort to prevent the interior from swelling and bursting the skin. Temperature control is a necessary step toward crack prevention. Cracks usually start at the boundaries of a concrete mass rather than at the interior. Boundaries are defined as the contact with bedrock and the surfaces of interior adits or chambers within the body of the mass, in addition to outside surfaces temporarily or permanently exposed to the atmosphere. The design problem for any large gravity dam is to provide large monoliths with wide bases that will be permanently free of cracks parallel with the axis. If cracks are allowed to form during the nonuniform conditions of construction, they will tend to grow as subsequent natural cooling takes place. Uncontrolled cracks can extend to bedrock and entirely through the dam. When a monolith splits from base to top, stability is threatened, stresses are altered and foundation loading becomes entirely different from that indicated in the design. The safety of the structure is threatened and life expectancy is greatly shortened.

The severity of climatic changes during construction, and afterward, should be considered with care in determining the method and extent of control for any project. The number of times the ambient air temperature undergoes a large differential in a short period during construction is important. A limit of 25 deg F, or more, differential of the ambient air temperature in a short period has been adopted as a reasonable measure of this condition. Table 1 compares the condition at the Detroit Dam site with that of other dams where such data are available.

Within the last 20 years, two methods of temperature control have been developed. One method consists mainly of cooling the concrete after placing by circulating cold water through embedded pipes. With this method, the

TABLE 1—AMBIENT AIR TEMPERATURE COMPARISON

Number of times the temperature differential exceeded 25 deg F during a short period	Dams					
	Clark Hill	John F. Kerr	Mt. Morris	Bull Shoals	Ft. Gibson	Detroit
	114*	135*	120*	143*	77*	175**

*One year elapsed time during construction.
**Average for one year during the construction period.

concrete is placed in blocks with joints in both vertical planes. Continuity is obtained later by grouting joints between the individual blocks of concrete. The interior concrete is sometimes cooled below the ultimate expected temperature to allow for persistent heating as the concrete continues to hydrate at later ages. Too-rapid cooling by this method sometimes results in cracking.

The other method depends mainly upon precooling the concrete aggregates, cement, and water before mixing and placing. It also takes advantage of other measures to limit the temperature rise and effects of rapid thermal change. The lowest practicable cement content is used in the interior mass; cements of limited heat liberation are specified, and construction procedures favorable to minimum expansion are adopted. Calculations are made of temperatures to be expected, and that combination of materials and procedures is specified which will promise most favorable temperature relations during construction and afterwards as the dam becomes adjusted to its ultimate condition. This method sometimes permits the adoption of designs without construction joints parallel to the long axis of the dam, and is in use by the Corps of Engineers, United States Army. Both this method and the pipe cooling method aim at producing, in the end, a monolithic structure with the maximum durability.

Detroit Dam design objectives

In the Detroit Dam special measures affecting both design and construction were directed at a low concrete placing temperature, limiting the temperature rise within the mass so as to avoid extremes at any localized zone, and toward maintaining a relatively uniform temperature rise during all seasons of the year. The objectives included production of a more flexible concrete, adequately strong, but less susceptible to cracking than more rigid high strength concrete, so compounded, placed, and controlled as to prevent, or at least to minimize, any tendency to crack. At the time this phase of the design was developed it was the largest known monolithic structure planned without vertical joints in the mass parallel with the long axis. The problem was to determine the controlling factors which would enable a dam of such size to be built, free of structural defects, without involving excessive cost.

Engineering manual provisions

The *Engineering Manual for Civil Works* issued in 1953 by the Office, Chief of Engineers, U. S. Army, is in use by the Corps of Engineers as a guide in the design and construction of large concrete structures. The following statements are direct quotations from this manual.

1. "General Considerations—The problem of control of temperature stresses is of concern principally in connection with the design and construction of gravity type concrete dams. It is a general concern, in connection with all types of massive concrete structures. The problem is caused by the temperature changes that take place throughout a structure due to heat generated as the hydration of the cement takes place. The problem is influenced by a number of factors, one of the most prominent being climatic temperature conditions at the site of the structure. The trend of temperature change follows a cycle. This can generally be defined in terms of three critical points in the cycle. They are: the initial or placing temperature, the maximum temperature, and the final or stable temperature. The interior of the dam will eventually reach a condition of almost constant (though not equal throughout) stable temperature." [The foregoing design and temperature control measures were in part made possible by the success of the Detroit Dam experience.]

2. "Methods of analysis of temperature changes in concrete structures have been published in technical literature and are generally available. The simplified analytical methods developed by Schmidt and Carlson are satisfactory for use in the solution of most of the problems related to investigation of temperature changes in a concrete structure."

3. "Cement—The use of Portland cement or a blend of Portland cement and a cement replacement having favorable heat characteristics is basic to the problem of temperature control. The use of Type II cement will, in general, be standard practice for Corps of Engineers construction. The use of Type IV cement will be given consideration only where a Type B (massive structure) is indicated."

4. "Protection—The temperature protection equipment, the curing water, and the removal of forms shall be handled in such a manner that the surface of the concrete will not be subjected to a temperature differential of more than 250° F."

5. "Control Methods—It is mandatory, from an engineering standpoint, to take every reasonable precaution to minimize cracking which may result from temperature changes."

6. "Forced Cooling—Under certain circumstances it may be necessary to utilize pipe cooling in combination with controlled placing temperatures."

DESIGN PROCEDURE FOR DETROIT DAM TEMPERATURE CONTROL

The first operation was to adopt those necessary conventional design and construction standards which were independent of the results of the thermal studies to be made. The entire dam was divided into monoliths or blocks 50 ft long. It was required that concrete be placed in horizontal layers or lifts 5 ft deep except where shallower lifts might be needed or justified. One of the early steps was to determine, by trial mixes, using the manufactured material proposed for the dam, the lowest cement factor that would produce a dense workable mix of necessary quality. The cement content was found to be about 0.6 bbl (235 lb) per cu yd of mixed concrete. The upstream and downstream faces of the dam (4-5 ft thick) are of a richer mix for increased durability.

Precalculation of temperatures

With laboratory results available, the next step was to calculate mass concrete temperatures for a variety of conditions. The principal factors to be dealt with in such a calculation are: diffusivity of the concrete; quantity

TABLE 2—SPECIFICATION REQUIREMENTS

Curve in Fig. 1	Cement type	Cement factor, lb per cu yd	Maximum placing temperature, F	Maximum heat of hydration,* cal per g	
				7 days	28 days
A	I	376	65	—	—
B	II	376	65	70	80
C	II	282	65	70	80
D	IV	235	50	60	70

*Heat of hydration used in this table is that resulting from 70 F temperature including immediate heat of hydration.

and heat-generating characteristics of the cement; methods of construction; and ambient air temperature at the site. The designer should not hesitate to select any of the standard types of portland cement available with due regard to cost. Those considered were Type I, Type II, and Type IV, as offered by the cement industry in this country. All cements considered produced concrete of adequate strength. The heat of hydration specified was not to exceed the values in Table 2. An admixture was required which would entrain air to help produce plastic concrete with a low cement content.

Fig. 2 shows how the precalculation of temperatures can help in choosing cement type and other factors. Four temperature curves represent temperature variation to be expected for the four conditions listed in Table 2. Conditions which remained constant for all four curves are: height of lift, rate of construction, ambient air temperature, and thermal properties of the aggregate. Variables are cement type, cement content, and placing temperature. It is not intended here to show the many calculations or curves necessary to illustrate the effect of each variable separately, but rather accurate

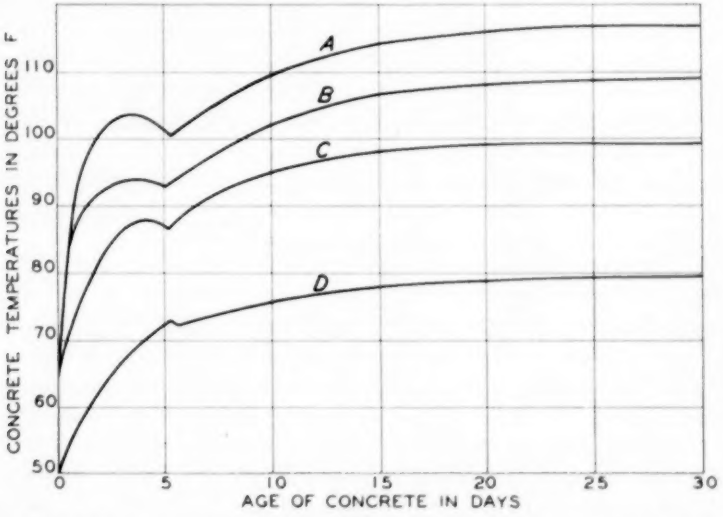


Fig. 2—Calculated concrete temperatures resulting from the consideration of data in Table 2

comparisons of results for illustration. Mechanical cooling of the concrete after placement by embedded pipes was not considered in these data.

Curve A shows that using 1 bbl (376 lb) of Type I cement per cu yd without precooling produces an average concrete temperature within the mass of 118 F at 28 days under average conditions. Curve B shows that using 1 bbl Type II cement per cu yd without precooling produces an average temperature of 109 F at 28 days under average conditions. Curve C shows that using 0.75 bbl Type II cement per cu yd of concrete (without cooling) produces an average concrete temperature of 99 F at 28 days, also under average conditions. A placing temperature of 65 F is known to be a fair average for this location and was used in these calculations. It has been determined that the interior mass of a large dam will eventually cool to approximately the mean annual temperature of the site, which at Detroit Dam approximates 50 F. Under these conditions the concrete would undergo a final reduction in temperature of slightly more than 49 deg. (Concrete C, Table 2). This condition was rejected as being unnecessarily severe and one which would not permit elimination of vertical joints parallel with the long axis of the dam, because the wide base would be ruptured due to cooling tensile stresses and possibly result in an unstable condition. Discussion later in this paper will shed more light on this important phase of the design.

Temperature calculations indicated that by cooling the fine and coarse aggregate and using refrigerated water in the mix concrete could be produced with a placing temperature of 50 F at all times. This assumed placing temperature is not necessarily the same as the mean annual temperature, although at the Detroit Dam location it did coincide. Curve D indicates that with the use of 0.6 bbl of Type IV cement and with cooled aggregate the concrete temperature within the mass in warm weather would be limited to 80 F at 28 days for what can be termed a normal construction schedule. This condition would mean an ultimate drop of 30 deg from the 28-day temperature through several years until the mean annual temperature should be reached. Not enough was then known about the ability of mass concrete to withstand thermal shrinkage without rupture, but such data as were available indicated that a slow drop of 30 deg would not damage the concrete. More details of this phase of design appear later in the paper.

Cooling varied with the season

Having determined, theoretically, that aggregate cooling could be depended on to produce 50 F concrete in warm weather, Fig. 3 was prepared to indicate the conditions throughout the seasonal temperature changes during construction. A plan capable of cooling materials to produce concrete placing temperature not over 50 F in the warmest month of the year would also produce concrete of a lower placing temperature during colder months. By additional calculations, it was discovered that Type II cement having a greater heat generating factor (see Table 2) could be utilized in colder months of the year when the placing temperature would be below 50 F and yet not

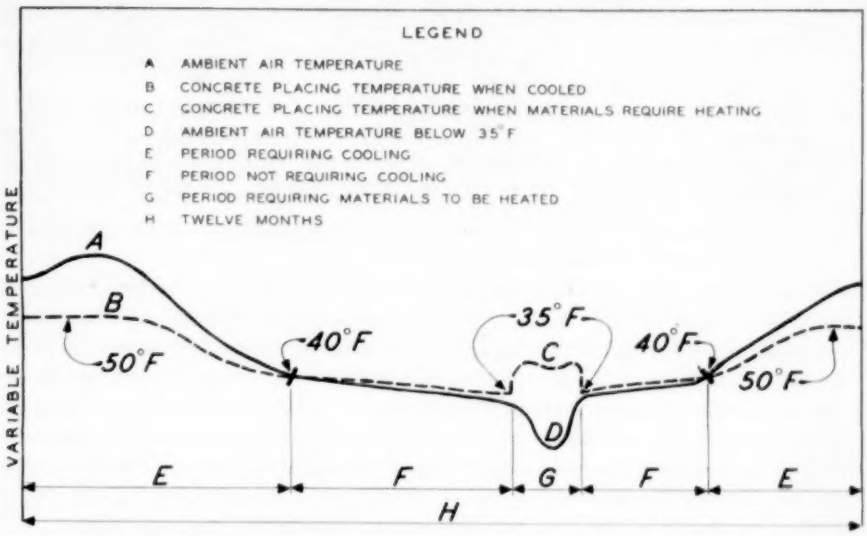


Fig. 3—Schematic diagram showing general problem of cooling (or heating) as required by project specifications

exceed the mass concrete temperature rise above that placed in summer months using Type IV cement with a placing temperature of 50 F. These data indicated that at the Detroit Dam Type II cement could be used in colder months of the year, its cost being $0.35 per bbl less than Type IV cement. Accordingly, the specifications for construction provided for the use of Type IV cement during the warmest months and Type II cement during the colder months, with a limitation on all concrete temperatures of 50 F in the forms 1 hr after being mixed. This related to the interior mass of the concrete with 0.60 bbl cement per cu yd of concrete. The exterior facing concrete was richer, with a higher rate of heat generation which was largely compensated for by a more rapid rate of heat loss by convection.

Construction methods specified

Complete control of construction procedures is vitally important if satisfactory results are to be obtained. The following seven items were specifically required for the Detroit Dam. All of these measures have not ordinarily been specified in similar structures. Taken together they constitute a complete coordinated control system to prevent thermal cracking. These special requirements were specified and closely adhered to during the more than 3 years of construction. They are as follows:

1. Type IV cement was specified for use from April 15 to October 15, Type II cement at all other times.
2. The placing temperatures of concrete were limited to a maximum of 50 F in the forms, with a minimum ambient air temperature of 35 F permitted without heating the concrete materials.

3. The height of leading blocks (monoliths) was limited to 25 ft above adjacent blocks.

4. Protection of transverse faces of blocks was required if exposed for more than 30 days before placing adjacent blocks.

5. Steel forms were required on the transverse faces of all blocks without the use of wood lagging.

6. Cement content of 0.6 bbl per cu yd of concrete was specified for the interior mass mix, together with the use of an air-entraining admixture. Outside or facing concrete required 1-1.1 bbl cement per cu yd of concrete.

7. Pipe cooling was specified in deep, irregular foundations and for the wide bases of certain blocks for a limited height.

Some of the above provisions require explanation for full understanding of their significance. The limit on placing temperatures throughout the changing season is illustrated in Fig. 3, which was the guide for layout of plant and for supervision by the resident engineer. Starting in the warm season the maximum concrete placing temperature must be 50 F or less, somewhat below the ambient air temperature. This required cooling of the aggregate. As cooler weather approached cooling of material was continued until the temperature of the concrete, if mixed of uncooled materials, would not exceed 40 F. At this time cooling was discontinued and not resumed again until the next approaching warm season when the temperature of the concrete would be 40 F if materials were not cooled. During the cold season, if the air temperature was 35 F or lower, materials were to be heated and other protective measures put into effect as provided in the specifications.

The provision that the height of lead blocks should not exceed 25 ft above adjacent blocks (Item 3) has for its purpose protection of the transverse faces of long exposed blocks from thermal shock because of rapidly falling air temperatures, which could rupture the young concrete. The interior concrete near the surface may be as warm as 75 F while the ambient air may be as low as 0 F. This condition is known to have caused transverse faces of monoliths in some dams to crack in numerous places.

The requirement that the transverse faces of all of such blocks be insulated if exposed longer than 30 days gives the same type of protection as that described above. In one case at the Detroit Dam it was necessary to extend one block above the adjacent block for about 1 year for construction reasons. At the end of that time, when the insulation was removed, no cracks could be seem—a good test for this provision.

Steel forms required on the transverse faces of all blocks were beneficial in preventing initial concrete surface temperatures from rising appreciably above air temperatures. It is often desirable to strip these forms at an early age. If thick wood forms were allowed, the removal and exposure of warm concrete to cold air would overstress the young surface concrete and cause vertical cracking parallel with the axis. Such initial surfaces of weakness form loci of later ruptures of serious proportions. The use of thin steel forms allows the fresh concrete to be chilled or cooled near the surface (during cold weather) while plastic and when stripped it experiences no appreciable change or thermal shock. Cracks might not result in serious consequence; on the other

hand, increasing use of steel forms where optional indicates that this important provision is not uneconomical. Particular care in many safeguards spells the difference between satisfactory and questionable results.

The wide base of this dam covers a number of depressions in the foundation rock, one of which extended more than 60 ft below the normal base of the dam. Others were almost that deep. Before Detroit Dam was constructed the method of cooling aggregate and mixing water to obtain a definite placing temperature had not successfully been carried out on a large scale; therefore it was decided to use mechanical cooling in addition to the use of cooled aggregate in the base of the deeper sections of the dam to insure satisfactory results. This reduced the maximum temperature range of the concrete that filled these deep recesses so that the broad base of the dam would rest on a uniform layer which would not later cool and contract irregularly.

Local cooling in this area was accomplished by circulating water at 38 F through embedded pipes. The pipe coils were placed parallel with the long axis of the dam, rather than normal to the axis as is sometimes done. This was intended to prevent any later leakage along the paths of the pipe where final cooling and shrinkage sometimes causes minute water passages to form. For the same reason the pipe coils were terminated several feet from the face of the dam.

Mechanical cooling permitted normal rates of concrete placement in these restricted foundation areas and at the same time resulted in adequately cooled concrete in contact with the foundation rock.

Mix proportioning

At the dam site the river water has an average pH value of 7.77; therefore, there was no problem requiring corrosion-resisting cement. Natural sand and gravel in the vicinity were found to be mildly alkali reactive. Fine and coarse aggregates were therefore manufactured from a local diorite to eliminate the possibility of alkali expansion and for economic reasons. Favorable results from freezing and thawing tests using the materials specified for the dam, indicated satisfactory durability. Density of the concrete is also satisfactory.

In addition to standard provisions for aggregate the specifications for the fine fractions differed somewhat from usual mass concrete proportions. The amount of fines required in the trial mixes to produce a workable concrete was found to be in excess of the usual percentage. It was found that 13 percent of minus-100 material, plus air entrainment, produced a workable mix with 0.6 bbl cement per cu yd as specified. This amount of fine material, some of which passed the 325 mesh, did not increase the water-cement ratio over that required by the coarser sand. The workability was greatly improved; in fact without more than the usual 6 percent of minus-100 material often specified for manufactured aggregate, the low cement content would not have been practicable. Resulting strengths were adequate, although with Type IV cement the low placing temperatures caused strengths to develop

TABLE 3—MIX PROPORTIONS AND AVERAGE STRENGTH OF DETROIT DAM CONCRETE

Item	Mass concrete			Structural concrete
	Interior	Face		
Maximum aggregate size, in.	6	6	3	1½
Fine aggregate, percent	26	26	31	36
Cement, lb per cu yd	235	376	423	517
Water, lb per cu yd	188	210	254	275
Fine aggregate smaller than 100 mesh, percent	13	13	13	13
Calculated air content, percent by volume of concrete containing aggregate less than 1½ in.	3 to 6	3 to 6	3 to 6	3 to 6
Water-cement ratio, by weight	0.8	0.56	0.60	0.53
Average compressive strength, psi:				
Type II cement, 28 days	1300	2740	2830	3410
Type II cement, 3 months	2210	4360	4338	4870
Type II cement, 6 months	2610	4720	4640	
Type II cement, 1 year	2560	4750		
Type IV cement, 28 days	820	2210		
Type IV cement, 3 months	2100	3260		
Type IV cement, 6 months	2480	4370		
Type IV cement, 1 year	2520	4870		

slowly, requiring greater care in form anchoring and removal. Table 3 gives some of the properties of this concrete. This table also distinguishes between the interior mass mix and the face mix. The amount of fines (minus-100) in the face mix was 13 percent, the same as for the interior. It was stated earlier that crushed material was utilized for this concrete. The rock sand manufactured by the rod mill produced an excess of fines, and it was therefore an economy to utilize the larger than usual amount of fine material.

Beam tests of mass concrete

Laboratory tests of Detroit Dam concrete in the form of unreinforced beams were conducted in which the deformation was measured at the time of failure by the use of Carlson strain meters embedded in the beams. The average deformation at failure of all beams thus tested was 76×10^{-6}. Of these 27 beams, about half were fast loaded and the remainder by what was termed slow loading. Duration of the latter test, however, was an average of but 54 hr, which for the purpose of this treatise must be termed fast loading. These tests indicated that the extensibility of concrete containing Type II cement, at time of failure was slightly greater than that of the concrete containing Type IV cement. However, when compared on the basis of relation of deformations to stress at time of failure, the Type IV cement concrete indicated a deformation, or extensibility, 26 percent greater than that containing Type II cement. Experimentally the coefficient of thermal expansion of concrete with Type II cement was found to be 4.27×10^{-6} per deg F and that with Type IV cement was 3.59×10^{-6}. The instantaneous modulus of elasticity of the Type IV cement concrete was less than that of the concrete in which Type II cement was used. Strains due to volume change resulting from reactions of mineral constituents of the aggregate, loss of moisture, loading conditions, and related causes will not be considered.

TEMPERATURE RECORD

Type IV cement was used during the warm weather months, and Type II cement was used from October 15 to April 15. In one dam monolith or block, these two cements were used alternately creating four zones throughout the height of more than 400 ft (Fig. 4). There will be no direct comparison of the merits of these two cements because (Fig. 2) lower placing temperatures prevailed in the cool months of the year when Type II cement was used. The temperature within the mass was measured by 162 Carlson electrical resistance thermometers in the monolith at various depths and near the boundaries and in the interior of the mass. Some 41 thermometers were installed in another block or monolith. Readings at a central location were transposed to temperatures. These readings were at first taken at 24-hr intervals. Later as the concrete temperature became more nearly stabilized they were taken less frequently. More than 3 years after the first concrete was placed, readings in exposed zones were at 1-week intervals and readings within the mass were at 30-day intervals. Embedded pipe cooling reduced the concrete temperature in Zone 1 and in the lower 20 ft of Zone 2. The river was diverted through a tunnel during construction which permitted the

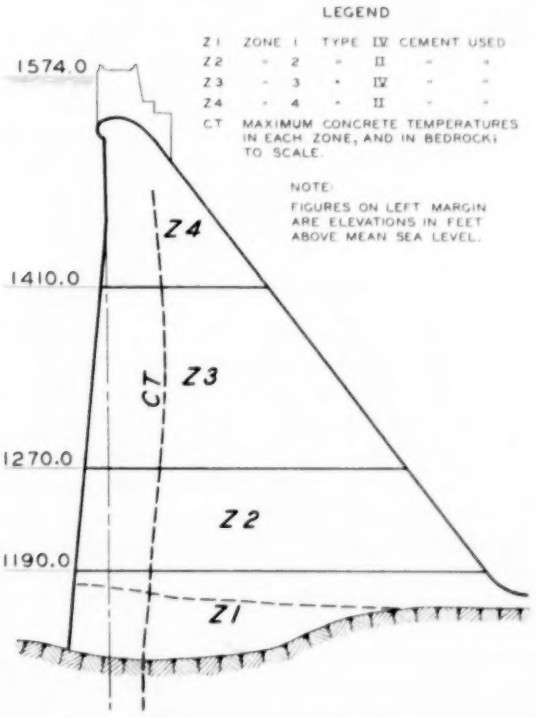


Fig. 4—Zones adopted for temperature and strain analysis

uniform rate of concrete placing favorable to a slow rate of temperature rise and decline, always desirable in a large mass.

Placing temperatures were generally maintained at 50 F maximum, although at the start of operations, before the plant was properly balanced and at one other time, mechanical difficulties caused placing temperatures to go as high as 56 F. The chart (Fig. 3) prepared to illustrate the periods and temperatures when cooling would be required proved to be a dependable guide. During the cooler seasons of the year, as the natural consequence of the methods of plant operations, placing temperatures ranged from 42 F to 50 F. Embedded pipe cooling of concrete in deep foundation irregularities and immediately above was successful using refrigerated water as a coolant. The temperatures at all thermometer stations within the mass (not including those near the surface) were averaged for each zone and single lines for each result were projected concurrently on a temperature-time scale (Fig. 5). There was less variation in the temperatures between thermometer stations in this case than is frequently experienced. This is believed to be due to the uniform limit on placing temperatures throughout the year. It is realized that averaging a number of temperature readings in a zone may seem to be oversimplification, but the thermometer station indicating the highest temperatures in the mass of the dam was found to be but 3 deg above the average temperature shown in *N*3, Fig. 5 (other zones similar). In the same way the temperature of one series of thermometers embedded in foundation rock, at variable distances below the rock surface, is indicated as a basis for discussion

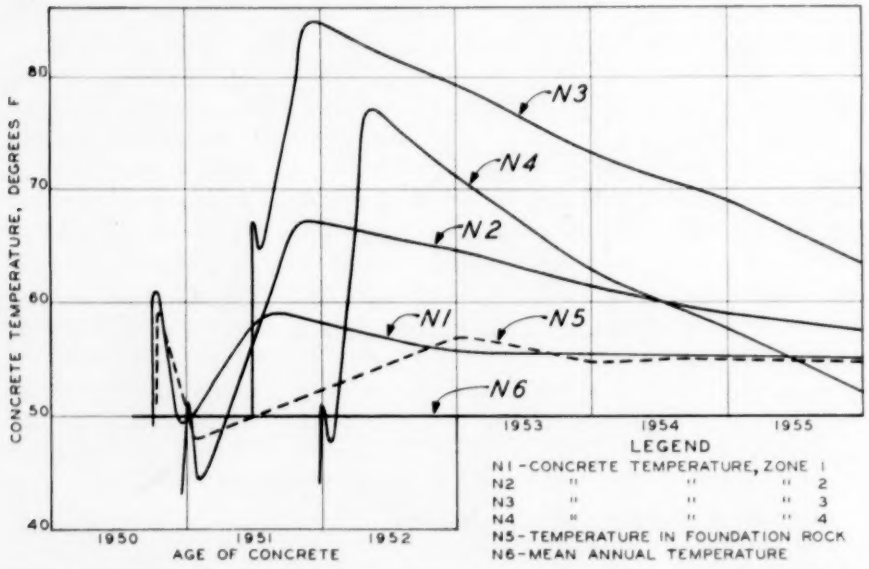


Fig. 5—Temperature records of mass concrete in Zones 1, 2, 3, 4, and foundation rock, Detroit Dam

to follow (see *N*5, Fig. 5). Concurrent with the temperature drop of any part of the mass near the foundation, there will tend to develop a tensile strain in the concrete and an accompanying compression in the rock. The converse of this is also true.

FOUNDATION CONDITIONS

It is believed that restraint of the foundation rock causes temperature strains at the base of the dam which are extended upward in the mass as later cooling takes place. If the temperature differentials are sufficiently low, the effect of this restraint is not objectionable. This comment refers to strains normal to the long axis of the dam, i.e., parallel with the flow of the river. If the base of dam were free to move with temperature changes there would be no problem of vertical ruptures forming parallel with the axis. The restraint of the concrete mass itself may sometimes cause surface cracking due to early cooling and even cyclical changes may have the same effect.

Since conditions at the base of a dam cannot be inspected (except during construction), the best approach to an analysis of conditions at and near the base must come from consideration of such basic data as temperature measurements and strain calculations resulting from accurately recorded temperatures. Importance of measured strains is not to be discounted but in the case of Detroit Dam strain meters were not installed. It is considered that dependable data as to the thermal coefficient of expansion of the concrete and accurate temperature measurements are of great importance in computing temperature strains. The temperature variations at the base of this dam are much less than in most large dams. The strain calculations will serve to substantiate an evident favorable condition resulting from small temperature differentials and the properties of the concrete.

TEMPERATURE STRAINS ANALYZED

It is not feasible to present a complete thermal stress analysis of this concrete. To do so would require the results of measured strains in the concrete and the foundation rock (such as may be obtained by use of Carlson strain meters), values for the modulus of elasticity, the coefficient of thermal expansion, and the creep characteristics of the concrete together with careful consideration of the amount of foundation restraint. These measurements were not all made. It is possible, however, to analyze the conditions in each zone by considering the volume strain resulting from temperature deformations and equating the results to the known ability of this concrete to resist being ruptured by the strains that have developed and which will result from ultimate cooling. The Tennessee Valley Authority has stated in this regard, "Many concretes will crack under tensile strains of 100×10^{-6}. Rapid loading may, in certain cases, cause concrete to crack at 80×10^{-6}, while the same concrete may exhibit ultimate strains of 160×10^{-6} if tension develops slowly."

When previously considering the temperature differential that would be acceptable, a drop of 49 deg was rejected as excessive. By adopting the conditions shown by Curve D, Fig. 2, a maximum final temperature drop of 30 deg could be expected. To compare with the TVA data mentioned above, strain for the assumed 30 F drop for Type II cement concrete was calculated as 128×10^{-6} and for Type IV cement concrete, 107×10^{-6}. Since the slow cooling in process at the base of the dam can certainly be classed as "developing tension slowly," these strains fall within the range indicated by the TVA statement. These data from the measured strains at Hiwassee Dam were not available at the time the Detroit plan was adopted.

The Hiwassee Dam studies by the Tennessee Valley Authority state further: "It must be concluded that from measurements and at lower elevations in the gravity dam 100 percent restraint will take place, at least during early ages of the concrete." The condition at the base is of greatest importance due to danger of cracks forming at this boundary. Some investigators assume that restraint becomes zero at $0.5H$ above the base. Others have stated that at $0.5H$ restraint may be 10 percent to 30 percent. In the case of Detroit Dam the small temperature rise at the base tends to extend the limit of foundation restraint upward. It will be assumed, therefore, that the restraint as affecting computed tensile and compressive strains was (and is) as follows: at the base of the dam, Zone 1, 100 percent restraint; Zone 2, $0.10H$ above the base, 70 percent restraint; Zone 3, $0.25H$ above the base, 45 percent restraint; Zone 4, $0.60H$ above the base, 0 restraint (these values at bottom of zones). None of these restraint values is completely certain, but available data favor them strongly.

Results of many strain meter installations in dams, as stated by R. W. Carlson, indicate little or no induced compressive stress in the first two or three days of temperature rise in lean concrete. For Detroit Dam the controlled strain is assumed to be zero at the time of the first maximum temperature which came at the age of about 3 days or less. Referring again to TVA data, "Norris Dam measurements indicated that as a first approximation the stress might be assumed zero at the maximum temperature."

In the following pages temperature strains in each of the four zones of the Detroit Dam will be analyzed at the time of maximum and minimum temperatures. These computed strains will then be compared with the known strain resistance of the Detroit Dam concrete from which conclusions can be drawn as to the safety of the design from that standpoint. Other stresses will not be considered in this presentation. Fig. 5 and Table 4 show concrete temperature variations for the four zones as well as for the rock below dam base.

Zone 1

Type IV cement was used in Zone 1, first in the concrete filling the deep recesses in the rock at the base of the dam and later in the entire foundation of the monoliths in which instruments were located and in other monoliths of equal depth. The rock was at a temperature approximating the placing temperature of the concrete. This is indicated by the temperature

TABLE 4—TEMPERATURE VARIATIONS WITH TIME

Zone	Placing temperature, F	First rise		First decline		Second rise		Rate of decline, deg F per month	
		Days	To temperature, F	Days	To temperature, F	Days	To temperature, F	First year	Last year
Foundation rock	51	6.0	58.8	73	48.0	2 yrs	56.9	0.33	0.05
1	49	3.0	61.0	39	49.0	220	59.3	0.21	0.05
2	43	2.5	51.0	28	44.3	276	67.0	0.21	0.14
3	50	3.0	67.0	6	64.8	155	85.9	0.62	0.37
4	44	2.5	50.9	4	47.5	124	77.0	0.75	0.38*

*Third year

and history of *N*5, Fig. 5. The temperatures at all thermometer stations in this zone were averaged to obtain these temperature curves. To assume zero strain at the first rise will be conservative, which means that computed tensile strains will be slightly on the high side. Tensile strains were computed using simple assumptions† for the first decline as shown by curve *N*1. Neglecting creep and assuming 100 percent restraint, the first decline of 12 deg produced a tensile strain of 43×10^{-6}. As a result of laboratory beam tests described above, Detroit Dam concrete is known to have an extensibility (to the point of breaking) of 76×10^{-6} when loaded rapidly. Without making allowance for increased extensibility for slow loading, the computed factor of safety against ruptures forming at that age is approximately 1.75. An assumption of slow loading and accompanying creep would greatly increase this factor of safety. The second rise of 10 deg to a temperature of 59.3 F, only 9 deg above the yearly average air temperature, indicates a compressive strain of 36×10^{-6}, almost equal to the initial tensile strain, resulting in a residual tensile strain of 7×10^{-6} at that time. Following this, the ultimate cooling now in progress, will again produce additional tensile strains of not over 32×10^{-6} or an ultimate strain of 39×10^{-6}.

A careful scrutiny of Fig. 5 discloses that not only did the rock temperature decline with the concrete temperature in Zone 1, but the rock temperature rose again with the second rise of the concrete during approximately the same elapsed time but not to as high a temperature as the concrete. The rock temperature continued to rise during the first 1½ years of the second decline of the concrete temperature at the end of which time the rock and the concrete temperatures were about the same. It might be argued that this condition was unfavorable, that the rock compressive strains should be added to the concrete tensile strain, or allowance made for compressive strain of the rock during the development of tensile strain in the Zone 1 concrete. If so, and if the rock strain was on about the same basis as the concrete, the sum of compressive strains in the rock and tensile strains in the concrete is about the same as strain resulting from a 9 deg temperature change, no more than has already been considered. Actually this means a tensile strain in the concrete developed by only a 3 deg drop in 1½ years. After this time when the rock and the concrete temperatures are about equal (rock 1 deg higher than the concrete) the decline in temperature for the next 3 years was nearly the same. At the end of that time the temperature of the rock and concrete (*N*1, *N*5, Fig. 5) was but 5 deg above the ultimate stable temperature. At the present rate it will require over 8 years for the foundation rock and concrete in the massive base of the dam to reach the final stable temperature together. This, like previous conclusions, is very favorable.

Conditions vary throughout the base of a large dam. There may be some conditions to partially offset the apparently favorable strains, but what has been stated indicates that all the concrete at the base of the dam will have undergone such relatively minor temperature strains that the factor of safety against vertical ruptures forming at any time is approximately 2.0 and greater, which is unusually large for this condition. The prime objective in the design assumption was therefore achieved in maintaining close control in this zone

†For a lengthy statement about tensile strains resulting from temperature changes, see: Clark, R. R., and Brown, H. E., Jr., "Portland-Pozzolan Cement as Used in the Bonneville Spillway Dam," ACI JOURNAL, Jan.-Feb. 1937, Proc. V. 33, pp. 183-221.

with the resulting tensile strain less than anticipated. It is considered important that the foundation of the dam, part natural rock and part concrete-filled recesses, should be thus sound and stable.

These values being so small, a complete stress analysis is unnecessary. If surface rock temperature had been much lower when concrete operations began, the temperature rise of the rock would undoubtedly have followed the same pattern as that shown on Fig. 5. A lower rock temperature would have chilled the fresh concrete while in a plastic condition, which would have been favorable, unless in the freezing range. Due to the very small temperature deformation at the Detroit Dam the result would have been satisfactory for any reasonable rock temperature at the start of concrete placing. The temperature of the rock when concrete operations start should, however, always be considered with care to insure a satisfactory adjustment of the massive structure to its foundation.

Zone 2

Zone 2 concrete was placed during the cooler season and therefore Type II cement was used. As noted in Fig. 5, placing temperatures were well below the required 50 F, although it should be noted that design assumptions took this factor into account. Zone 2 is approximately $0.10H$ above the base of the dam; therefore, the foundation restraint producing tensile strains was assumed to be 70 percent of that resulting from the product of the temperature change times the coefficient of expansion. Again assuming a no-strain condition at the first rapid rise that took place at 2½ days, the first decline of 7 deg produced a tensile strain of 21×10^{-6}. On the same basis as in Zone 1, a factor of safety of 3.6 exists as far as danger of rupture is concerned. If conditions in this zone were considered to be modified by the effect of the zone below, it is probable that the actual tensile strain was less than that computed, thus increasing the factor of safety. The second rise of 22.7 deg introduced a compressive strain of 68×10^{-6}, minus the existing tensile strain, or a resultant compressive strain of 47×10^{-6}, which at the present time is being relaxed by the slow final cooling through 17 deg. With the same assumptions as before, this period of cooling will result in a final tensile strain of only 4×10^{-6}.

Concrete does not strictly obey the rules of elasticity, plastic flow, delayed action, and deformation. It may be considered, therefore, (not by the author) that as the second temperature rise to 67 F produced compressive forces, creep of the young concrete may have caused the volume change to nullify part of this induced elastic compressive strain, and that during the final cooling, now in progress, a larger tensile strain may be induced. If that be true, and if the accompanying probability of creep and tension during this cooling also be ignored, then the rise through 22.7 deg and the fall of 17 deg (assuming a 50 percent creep) cannot produce a tensile strain of more than 38×10^{-6}, which again yields a factor of safety of about 2.0. Ignoring the last remark, these minor temperature deformations will at no time result in tensile strains of more than 27 percent of the known extensibility of the Detroit Dam concrete, even if rapidly loaded. If, as had been stated, slowly developed tensile strain is considered, the factor of safety against cracking in this zone is even more favorable. The fact that the temperature in Zone 2 was rising at the same time it was rising in Zone 1 was favorable, as was the fact that the lower 20 ft of Zone 2 was mechanically cooled to somewhat lower temperatures than the remainder, or the average, of Zone 2. If an accurate stress analysis was possible this condition would be very favorable as far as tensile stress is concerned.

Zone 3

In Zone 3 Type IV cement was used. The concrete temperature rose to 67 F in 3 days. There was a slight drop and again the temperature rose to approximately 86 F at an average age of 5 months. This was within the assumption of not over a 30 deg rise in 28 days, but in the analysis of strain conditions the total rise will be considered (rather than the rise that took place in 30 days). The higher than average temperature which occurred in this zone was caused partly by mechanical difficulties in the sand cooling equipment. Four thermometer stations placed in the 5-ft lift concerned (mechanical difficulties) raised the recorded average

temperature of the 13 thermometer stations in this zone by at least 3 deg, although due to the season of the year the placing temperature was 50 F. A conservative assumption as before will be adopted and strains computed for the drop of 2.2 deg followed by the rise to 86 F. Again referring to the explanation of foundation restraint, the base of Zone 3 being approximately $0.25H$ above the base of the dam, the percentage of restraint will be assumed to be 45. Since Type IV cement was used in this concrete, the computed tensile strain was 4×10^{-6}, followed by a compressive strain of 34×10^{-6}, a resultant compressive strain of 30×10^{-6}. The subsequent cooling, now more than half completed, will produce a computed tensile strain of 58×10^{-6}, or a final resultant tensile strain of 28×10^{-6}. This means a factor of safety of 2.7 considered as fast loading. To assume that this concrete will not rupture within the range of 160×10^{-6} (according to TVA statements) a factor of safety in this respect of 5.7 can be said to exist when final stable temperatures have been reached.

Zone 4

The conditions of Zone 4 with Type II cement, are of less importance than the other zones because temperature variations are less than in Zone 3, immediately below, and being near the top of a large cantilever, the strains would seem to be greatly reduced. It was stated under Zone 1 discussion that it was assumed that at $0.5H$ restraint ceased to develop tension. The base of this zone being at approximately $0.6H$ above the base of the dam would appear to be unaffected by the restraint as far as tension is concerned. The first rise was completed in 2.5 days (compressive strain not considered), and the first decline of 3.4 deg took place in 47 days which was followed by a rise of 29.5 deg. After this a final decline of 27 deg will take place somewhat more rapidly than in Zone 3 where the mass is greater and the rate of cooling therefore is slower. At the end of 1955 the temperature in this zone was but 2 deg above normal. While no foundation restraint exists at this level, the restraint of a large mass may develop tensile strain due to temperature changes. In this case these strains would be very small. The temperature decline in Zone 3 during the temperature rise in Zone 4 was so slight that the otherwise adverse effect can be ignored. The factor of safety against cracking in this zone is favorable. This condition was anticipated in the design assumption and the induced compression in Zone 4 may not be entirely relieved with final cooling except for the surface boundaries which will be discussed separately.

Temperature gradients

Interpretations of the results of the Carlson electrical thermometer began early in the life of the dam as soon as sufficient readings were available. Results of these studies disclosed what undoubtedly contributed to the absence of cracks in the mass concrete. That was the slow rate of temperature decline which is desirable in the early ages when the concrete had not attained its full tensile strength. This was influenced by the favorable temperature gradient. The maximum rate of decline in any thermometer station was found to be 1.2 deg per day in Zone 1 where the average for the zone was 0.3 deg per day during the first decline. In Zone 2 the maximum rate was 1.0 deg per day with an average of 0.26 deg per day during the first decline. In Zone 3 the average rate was 0.37 deg per day. In Zone 4 the average rate was 0.75 deg per day during the first decline in temperature. It is believed that these rates are exceptionally low as compared with other large dams. The temperature gradient from interior to surface was correspondingly low, especially in the lower zone where it was not more than 1.0 deg per ft.

Surface boundaries

Consideration of the permanently exposed concrete surfaces of the mass concrete deserves attention, although not of structural significance comparable with the interior of the mass. The solar orientation of the structure is important since it has been shown that concrete as little as 1 in. below the surface may develop a temperature considerably above the ambient atmosphere of an average day. The Detroit Dam faces west; that is, the spillway and non-overflow faces slope downward toward the west and therefore are in an intermediate position, receiving less than the maximum amount of solar heat. The interior stable temperatures of a large dam are not uniform. Solar heat will be absorbed at the downstream face during warm weather and given off during colder weather. The average of four thermometer stations 6 in. below the surface of the concrete in Zones 1, 2 and 3 shows a maximum temperature range of 47 deg at an age of 6 months. This condition will improve as the interior of the mass cools, although before, or after, the interior has reached a stable temperature, an ambient air temperature of 0 F is possible at this location. In its Hiwassee Dam studies the TVA states that "in actual construction the effect of restraint of the surface is usually about 50 percent, indicating that surface concrete may actually sustain as much as 60 F under favorable conditions." Measurements of strain at the Shasta Dam as published by the Bureau of Reclamation include the following: "While interior is increasing the exterior is developing tensile stress. Measurements parallel with surface at Shasta Dam, show that at 2 ft inside the surface, annual cyclical stress variations as high as 800 psi exist." These are compressive stresses. If a differential of 60 deg should occur at Detroit Dam and if 50 percent restraint is assumed, the computed tensile strain would be approximately 120×10^{-6} without considering the effect of creep. While such a change can only occur in the colder seasons, a severe drop can come quickly and therefore more nearly represent rapid loading than otherwise; this precludes creep from greatly modifying results. As the dam cools, a slow transfer of compressive stresses tends to introduce vertical compressive forces in the downstream face which may or may not equal the tension that develops from temperature differentials. Tensile strains are of a triaxial nature and only those tending to act vertically would be opposed by load stress. Perhaps this is why some dams crack vertically on exposed faces as reported at the Vermunt Dam. There is therefore no assurance that surface cracking will not occur under the most unfavorable conditions at Detroit Dam. Such surface cracking was in evidence at some interior boundaries. In open adits and conduits in Zone 2, where cold winds were allowed to pass through these chimney-like openings during construction, temperature extremes were apparently greater than any recorded by thermometers. A few cracks in the order of 0.02 to 0.05 in. wide were observed. None are leaking. Cores taken in some of these cracks give an average depth of 4.3 ft. None of these cracks has any structural significance.

COST OF MASS CONCRETE CONTROL

Cost of mass concrete control in this project was made a matter of engineering study after the work had been largely completed. Rapid cooling of young concrete is to be avoided, especially if a temperature decline of as much as 30 deg must be considered. The maximum decline in the early life of the concrete in any of the zones was but 12 deg. It is not believed that any other method would have yielded equal results from the standpoint of favorable temperature differentials.

The problem therefore resolved itself into the question, "Do results obtained justify the costs and do results fulfill the designer's expectations?"

A careful estimate was made of the actual cost of cooling the Detroit Dam concrete by the method used, namely inundation of aggregates, cooling the sand, and using 38 F refrigerated water in the mix to obtain the required placing temperature of 50 F. This cost was found to be 56 cents per cu yd. The additional cooling by embedded pipes at the base of the dam amounted to approximately 74 cents per cu yd for the concrete cooled by that means. Since this meant double cooling for the concrete so treated the total cost of cooling by embedded pipes when prorated over the total dam volume amounted to 7 cents per cu yd, or a total cost for all cooling of approximately 63 cents per cu yd, about 4 percent of the total cost of mass concrete. It is considered that this is a satisfactory cost. The bid price for pipe cooling was approximately 74 cents per cu yd for the concrete cooled, and if applied to the whole dam might have been higher based on other similar cost data. So far as is known there were no delays or construction difficulties encountered because of the method chosen for mass concrete control.

RESULTS REALIZED

The ultimate test of a proper design of any structure is, "Have anticipated results been obtained? Do results agree with the designer's concept in all respects?" The answer to these questions at the Detroit Dam has been highly satisfactory. Inspection in mid-1956, with the reservoir full, 3 years after completion disclosed no outside surface defects to be detected any place by use of binoculars. A report during construction indicated the presence of a horizontal crack in the downstream surface, but no indication of such a defect can now be seen.

An inspection of all interior exposed surfaces discloses equally satisfactory conditions. Small surface cracks were reported previously in an adit, conduit, and valve chamber due to thermal shock during construction. These cracks cannot now be detected in all cases and those that can be seen are only 0.02 to 0.04 in. wide and are not continuous. This was to be expected since they were shown to be surface cracks only and did not create zones of weakness; therefore continued cooling has not caused any change in the surface appearance. An adit close to the left abutment originally contained vertical cracks on the abutment side due to the thin concrete wall separating the adit from

the rock. Some leakage could be seen at one time. This condition has now corrected itself, the only remaining evidence being stained surfaces and the usual deposits on walls and floor to a limited extent, but the adit is now dry.

Elsewhere in the interior of the dam, surface cracking or crazing could be seen in a few limited areas. Repair work and construction changes in progress at the time of this inspection disclosed the condition of this concrete where the surface had been cut away to a depth of a few inches. The crazing or small pattern cracking when visible on the original surface could not be seen on adjoining areas where the concrete surface had been removed, thus indicating the shallow depth of the crazing. No leakage stains can be seen at any of these small areas.

Leakage in the foundation area is not great. Many of the foundation drains, drilled into rock after foundation grouting, are leaking or have leaked slightly. One vertical drain placed in the contraction joint near the upstream face was leaking visibly. A small amount of seepage is coming from two of the contraction joints, no doubt due to improper connection of copper waterstop sections. Leakage through the mass concrete was formerly reported visible in five places. Now only one of these areas is leaking visibly. This condition indicates a small imperfection in the surface of the gallery at low elevation. No record exists as to any change from the normal construction but on viewing the conditions it appeared possible that one of the drains from the contraction joint had possibly been led into the drainage gallery inadvertently and had later been removed, or partially removed, leaving a defect in the mass concrete, through which a very small flow of water was emerging from the mass concrete. The amount of water is of no moment; its presence does not detract from the otherwise satisfactory condition of the concrete. Another area at the lowest level of the drainage gallery was damp for a few feet on one wall with stains indicating former leakage, now healed.

The report of this date indicates nothing that suggests deterioration of the concrete; in fact, all phases including the question of leakage, have improved during the three years since the last inspection was made. There was a certain amount of condensation, especially in the lower levels of the dam at the time of this inspection, which in some sections of the United States is a problem of maintenance. It was not difficult, however, to distinguish between dampness resulting from condensation as compared with other types of moisture inside the dam. The temperature inside the dam was at or below the mean annual, indicating a reduction in air temperature due to evaporation on concrete surfaces. With this assumption the temperature within the mass is in agreement with those measured by the thermometer installation.

During the work of placing mass concrete, constant inspection of newly stripped surfaces of monoliths, where visible above adjoining monoliths, disclosed no surface cracks at any time. Likewise all horizontal surfaces were

carefully inspected after cleaning and before the next lift was placed, with no cracks visible at any of the more than 1200 lifts during the 40 months of concrete placing.

SUMMARY

The most important result of the concrete control measures adopted is the uniformly stable condition at the base of the dam. The concrete base and foundation rock have experienced only slight thermal differentials. The concrete temperature rise did not equal the design assumptions in any zone. The maximum rise was 22 deg to a temperature of 72 F in the first 30 days. The analysis of thermal strain in the concrete indicates a substantial factor of safety in respect to the danger of structural ruptures forming at the base (or in any zone). It is not unreasonable to assume that the massive base and foundation rock supporting the dam, being in a near stable condition, served to prestress the zones above and insure the monolithic character throughout the dam, the proper objective of all design studies for large dams.

The seasonal use of the two cements, each of limited heat liberation, the use of a low cement content in the mass concrete, precooling aggregate and mixing water, aided by mechanical cooling of some of the concrete, and the adoption of certain unique construction techniques produced the desired results without involving construction difficulties or excessive costs.

Careful inspection of all surfaces as construction progressed disclosed no cracking on ends or surfaces of blocks. Some interior boundaries were subjected to thermal shock which resulted in a few shallow cracks. The density of the concrete as evidenced by lack of leakage throughout the mass is satisfactory. It is possible that downstream surfaces exposed to cyclical temperature changes may still be subject to cracking. This is not likely to be serious.

It may be assumed by some that the results achieved go beyond that required for "safe results." This is a false assumption. Observations through many years indicate that the Detroit Dam type of mass concrete control has not been used successfully in many cases and that the engineer is often inclined to "hope for the best," especially if he thinks that proper control will "cost money."

The economy of this method of concrete control was demonstrated by results. A total cooling cost of 63 cents per cu yd of mass concrete appears less than for any other known means which would have produced acceptable results. The satisfactory quality of the Detroit Dam concrete and its relative freedom from cracks may be attributed to careful planning in the design stage and to inclusion of an accurately prepared specification in the contract documents. All these provisions were faithfully carried out during the constantly changing conditions of construction.

ACKNOWLEDGMENTS

At the time these design controls were adopted, Roy W. Carlson was consultant to the Portland District, U. S. Army Corps of Engineers, and gave much valuable assistance. Some quotations from Tennessee Valley Authority and U. S. Bureau of Reclamation publications have been utilized where they seemed especially applicable.

The views expressed are solely those of the author and do not necessarily represent the official views of the Corps of Engineers on the subject.

Discussion of this paper should reach ACI headquarters in triplicate by Sept. 1, 1957, for publication in the Part 2, December 1957 JOURNAL.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Construction

Conoidal roof

L. C. Howitt, *Prefabrication* (London), V. 4, No. 39, Jan. 1957, pp. 103-105

Describes an English factory roof in which the typical bay is 30 x 100 ft. Prestressed bolstering trusses of 100-ft span carry prestressed purlins at 6-ft centers, running from the top of each arch to the tie of the next arch producing a conoid shape. The roof covering on the purlins is a fiber material.

Chimneys for Marina and Ringsend power stations

James Daly, *Bulletin*, Institution of Civil Engineers of Ireland (Dublin), *Transactions* V. 83, No. 4, Apr. 1957, pp. 89-112

Describes the design and construction of two 250 ft high reinforced concrete chimneys. Includes details on structural design, protective finish, description of grit collectors, specifications for materials and protective treatments used, and a bibliography containing 23 references.

Design

Skyscraper of reinforced concrete (Le gratte-ciel en béton armé)

V. Eroscuchi, Edition Dunod, Paris, 1956, 94 pp., 1480 F

Reviewed by Alexander M. Turitzin

A French translation of the Italian publication entitled *Il Grattacielo in Cemento Armato* which first appeared in Genoa, Italy. Like *The Reinforced Concrete Building*, a previous publication by the same author, the purpose of this book is to outline design procedure for a tall reinforced concrete building. After a brief discussion of the economics for erecting a tall building, and the presentation of several plans and layouts of office and apartment buildings, the author presents a complete analysis and design of a 12-story apartment house.

Bending moments in columns and girders are determined by the method of Mario Baroni, and by the method of Hardy Cross. The values obtained from both methods are compared with each other. The author believes that a practical height limit for a reinforced concrete building is about 30 stories. Above that height, columns become so large as to occupy too much valuable space in the lower floors of the building. The book is clearly written, with numerous explanatory notes, sketches, and diagrams. It can well serve as a guide for a design of a tall reinforced concrete building, and as such, prove a valuable addition to the library of a practical engineer.

Influence of hypotheses and methods in reinforced and prestressed concrete calculations (Influence des hypothèses et méthodes dans les calculs de béton armé et de béton précontraint)

J. Barets, *Annales de l'Institut Technique du Batiment et des Travaux Publics* (Supplement) (Paris), No. 106, Oct. 1956, pp. 857-874

Reviewed by Edmund A. Pratt

Inadequacies of the elastic theory, errors resulting from the use of simplifying hypotheses, influence of shrinkage, and variability of safety factors are considered. Limitations of the elastic theory as a basis of design for a material subject to plastic flow are pointed out. The effect of creep on modulus of elasticity of concrete, and the effect of cracks in concrete on the apparent modulus of elasticity of the steel, are discussed. An example is given of a continuous beam in

A part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53. Address P.O. Box 4754, Redford Station, Detroit 19, Mich. Where the English title only is given in a review, the book or article reviewed is in English. If it is followed by a foreign title the work reviewed is in that language. In those cases where the foreign title cannot conveniently be set in type or is not available, the language of the original article is indicated in parentheses following the English title. Copies of articles or books reviewed are **not** available through ACI. Available addresses of publishers are listed in the June "Current Reviews" each year. In most cases ACI can furnish addresses of publications added later.

For those members that cut apart this section for pasting on cards for card indexes, a limited number of complimentary reprints of the 'Current Reviews" section are available from ACI headquarters on request.

which the aggregate of these effects increases moment at midspan by 30 percent and reduces the moment over supports by 14 percent. Assumption of a constant moment of inertia in the design of continuous structures is shown, in an example, to introduce errors sufficient to increase the midspan moment by 21 percent and decrease the moment over supports by 12 percent. The effect of shrinkage is considered and an example given in which shrinkage causes an increase in midspan moment of 14 percent. Summing up these effects the author concludes that the resulting total increase in midspan moment may be 65 percent and the decrease in negative moment 35 percent.

The variability of safety factors owing to lack of precision of calculations, variable quality of materials and constructional operations, and varying relative proportions of dead and live loadings, is discussed. The theory of probability is employed to show the possible variations in average strength of test specimens when a small number of specimens is involved, under assumed chances (99 in 100) of the test results being truly representative. Normal (Gaussian) frequency distribution of concrete compression test results is assumed and it is stated that a normal frequency distribution of elongations of identically stressed cables in prestressed concrete girders was also found from a study of 2700 test results. For elongations the coefficient of variation (standard deviation divided by mean) ranged from 4 percent to 10 percent, but was comparatively constant for any one job.

The author concludes that mathematical theories alone are insufficient for adequate design of reinforced concrete, which in a measure adapts itself to load conditions, and that at present such design is an art rather than a science. Paper is followed by a discussion.

Virtual work and the moment distribution method

B. G. NEAL, *Engineering* (London), V. 183, No. 4740, Jan. 11, 1957, pp. 47-50

Reviewed by ARON L. MIRSKY

Equilibrium equations required for analysis of sway are derived from geometry of sway movements by use of principle of virtual work. A one-span gable-type portal frame is used as a worked example.

Materials

Use of fly ash and similar materials in concrete

ANGUS A. FULTON and WILLIAM T. MARSHALL, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 6, Nov. 1956, pp. 714-730

Reviewed by ARON L. MIRSKY

North of Scotland Hydro-Electric Board was interested in the use of fly ash to conserve cement, provided resulting concrete was impermeable, in view of aggressive character of Scottish waters. Besides several pages of tabulated test results of concrete containing fly ash, paper reviews examples of use of such concrete in Canada and the United States. Conclusion: replacement of up to 20 percent of cement by fly ash is permissible, provided concrete is not required to withstand its working load until some time after placing.

Thermal coefficient of expansion of concrete aggregates

WILLIAM LERCH, *ASTM Bulletin*, No. 219, Jan. 1957, pp. 36-40

A report by Subcommittee III-b on Methods of Testing for Volume Change of Concrete and Concrete Aggregates of ASTM Committee C-9 on Concrete and Concrete Aggregates. Describes and discusses results on various typical concrete aggregates of a dilatometer method of measuring cubical dilation due to thermal changes for the determination of a mean thermal coefficient of expansion of both fine and coarse aggregates. Test method is completely described in appendix.

Some notes on hydrophobic or waterproofed cement

Civil Engineering and Public Works Review (London), V. 52, No. 609, Mar. 1957, pp. 330-331

A hydrophobic cement consisting of portland cement to which an additive has been introduced, giving the grains a protective covering which inhibits hydration in the absence of mechanical abrasion, is claimed to possess greatly improved stability even when stored under damp conditions. When subjected to abrasion, as in the ordinary concrete mixing process, protective film over the cement particles breaks down and the hydration process proceeds normally.

Precast Concrete

New trends in the industrialization of building (Tendences nouvelles de l'industrialisation du batiment)

JEAN MEUNIER, *Annales de l'Institut Technique du Batiment et des Travaux Publics* (Supplement) (Paris), No. 107, Nov. 1956, pp. 962-976
Reviewed by EDMUND A. PRATT

Describes the Roger Marie process of manufacturing hollow precast lightweight concrete panels and slabs for walls, partitions, floors, and ceilings. Pozzolan concrete is employed and surface coatings of plaster, cement mortar, or thin (about $\frac{1}{8}$ in.) tiles are applied at the time of casting. Tapered tubular forms are employed to produce cavities the width of the panel, into which steam is introduced at atmospheric pressure for curing, the panel being at the same time placed in a steam oven (atmospheric pressure) and tilted to a 30 deg angle to drain off the condensate. Casting and finishing processes and assembly line equipment are described, and the thermal properties of the product discussed.

Precast concrete comes of age

J. J. POLIVKA, *Consulting Engineer*, V. 9, No. 1, Jan. 1957, pp. 58-63
Reviewed by ARON L. MIRSKY

Major portion of article reviews American precast concrete structures constructed in the second decade of this century, some 40-odd years ago. More recent structures, including the proposed Polivka-Wright design for the second Transbay Crossing in San Francisco, the "Butterfly Wing" bridge, are much more briefly noted at the end.

Precast frame for suburban post office

Constructional Review (Sydney), V. 30, No. 2, Feb. 1957, pp. 23, 35

The Australian postal department has standardized a precast concrete frame for suburban post offices which are to have areas of 4000 sq ft, and two-story roof levels, providing clerestory lighting to the center of the building. Reusable forms were employed for the precast frame so that the plan could be reproduced in several localities where there is a backlog of need for small new post offices.

Lightweight masonry units (in Dutch)

IR. J. STELLINGWERFF, *Cement* (Amsterdam), V. 8, No. 13-14, Feb. 1956, pp. 305-310
Reviewed by JOHN W. T. VAN ERP

Only a few years ago concrete masonry units were rarely used instead of brick. Now, however, lightweight units are increasingly used in the Netherlands for wall and floor units, mainly due to the acute shortage of brick, lumber, etc. Among the materials used—cement, sand and coarse aggregates, the last needs most care in testing and inspection due to its varying properties. Slag as an aggregate has been a source of many troubles; it often needs preliminary chemical treatment like some other materials. A set of specifications and standards has been set up for these units cooperatively by the building departments of the biggest cities. It includes limits for permissible shrinkage.

Technical school at Amsterdam (Christelijk Technische school te A.)

C. DE GEUS and J. B. INGERWESEN, *Cement* (Amsterdam), V. 8, No. 19-20, Aug. 1956, pp. 449-457
Reviewed by JOHN W. T. VAN ERP

This building is particularly remarkable through its almost exclusive use of prefabricated concrete parts, and presents a striking modern architectural aspect. The building is designed on a module of 3.50 m determined by use requirements. Concrete as used meets very rigid specifications. Most interior and exterior concrete has received no additional finish and is left as is after stripping forms. Some of the prefabricated shockcrete elements used are: stair treads, stringers and hand railing; monolithic window frames; facade revetment of thin sandblasted slabs fixed to the concrete frame by means of bronze inserts and bolts; and window and door sills. Some decorative panels in the hallways are made by applying templates on the insides of the forms, thereby producing ornament recessed in the concrete.

Precast reinforced concrete grain silos (Silos a blé)

C. REICHA, *La Technique Moderne Construction* (Paris), V. 11, No. 5, May 1956, pp. 157-160
Reviewed by ALEXANDER M. TURITZIN

Novel type of construction for reinforced concrete grain silos of large capacity has been tried in Algiers. The project comprised six

silos 18 m in diameter and 37 m high. Each silo was subdivided by vertical walls into 12 triangular sections, with interior walls radiating from the center of the silo and exterior walls forming the straight lines of a polygon inscribed within the circle of 18 m diameter. The center of the silo consisted of a precast vertical cylindrical shaft. The vertical walls were made up of precast panels 1 m high and varying in thickness from 8 to 14 cm. The size being determined by the carrying capacity of cranes. The vertical intersections of walls along the exterior perimeter of the silo, and those along the circumference of interior shaft, were made of cast-in-place concrete, and well keyed into each other by reinforcing steel.

Particular attention was given to watertightness of joints. A bitumen-impregnated cord, 18 mm diameter, was placed at the center of all horizontal joints of exterior walls. All exterior sides of joints were caulked with mastic, and all interior sides of joints were pointed up.

Compared with the conventional silo construction, this type of design effected considerable saving in cost. However, it was felt that thorough planning of precasting and erection operations, plus rigid field supervision, were largely responsible for the saving.

Concrete paving blocks (Betonnen straatsteenen)

A. A. Van der Vlist, *Cement* (Amsterdam), V. 8, No. 19-20, Aug. 1956, pp. 461-468

Reviewed by John W. T. Van Erp

Production of these blocks is increasing rapidly in a country where brick is extensively used as a paving material. Some advantages of concrete paving blocks are: uniform and accurate dimensions; possibility of intricate shapes, for instance, for interlocking paving patterns; possibility of many different colors for traffic markers, etc., and great resistance to abrasion, maintaining anti-slip qualities longer than brick pavement.

The Ministry of Transport has established preliminary standards. The concrete blocks are preferably laid on sand bed which has been compacted by vibration. Installation specifications depend on existing soil conditions.

Pier sections cast and set with improved techniques

Contractors and Engineers, V. 54, No. 4, Apr. 1957, pp. 58-61

Precasting was utilized for bridge piers for a bridge over the Columbia River at Portland, Ore. Bases, bell bottoms, and pier shafts were precast in sections which weighed up to 70 tons. Prefabricated sections were set and filled to make each pier a solid mass.

Precast corrugated concrete roofs

Concrete and Constructional Engineering (London), V. 52, No. 2, Feb. 1957, pp. 79-81

Describes a method of constructing precast multiple barrel thin-shell concrete roofs. The method is similar to that offered in the United States by the Vacuum Concrete Corp. except that mechanical handling equipment is used. Completed assembly, however, is designed as a two- or three-hinged arch, rather than as a shell.

Properties of Concrete

Heat flow in the steam curing of concrete products

Allan Dawson Ross, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 6, Nov. 1956, pp. 695-702

Reviewed by Aron L. Mirsky

Beneficial effect of steam curing is shown to be a function of size of product treated: integral of temperature and time for a given mode of treatment increases somewhat with increase in size. In a complete cycle of steam curing, with uniform cooling to original temperatures, large blocks attain a somewhat greater maturity than small test cubes, and latter are therefore representative of the products. Effect of heat of hydration of ordinary portland cement is of secondary importance.

Gas and foam concrete (Gas en schuimbeton)

J. J. Smeele, *Cement* (Amsterdam), V. 8, No. 19-20, Aug. 1956, pp. 487-489

Reviewed by John W. T. Van Erp

Describes the research program carried out by the German Reinforced Concrete Institute, mainly on behavior of reinforcing steel, such as its bond and slip values and corrosion protection, in lightweight concretes. Also re-

search on effect of moisture on volume of finished product. Concretes with high cement contents were found to show greatest amount of shrinkage during hardening, while concretes with little cement and some lime showed smallest amount of shrinkage. Compressive strength was investigated for various sizes of aggregate, also for concrete made by the intrusion method, where aggregates including very large boulders were used. This last method proved to be economical as to cement use, at the same time producing concrete of considerable compressive strength.

No-fines concrete as a structural material

RONALD H. MACINTOSH, JOHN D. BOLTON, and COLIN H. D. MUIR, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 6, Nov. 1956, pp. 677-694

Reviewed by ARON L. MIRSKY

No-fines or cellular concrete, a direct result of lack of certain material (face brick, etc.) and of superabundance of other material (brick rubble) in some portions of Europe, is an excellent illustration of the familial relationship of necessity and invention. Paper discusses its advantages and disadvantages, its effect on structural design, its preparation, placing, and protection from the elements, and its effect on building methods, as compared to traditional method. Data cited are primarily from construction of two- and three-story houses in Scotland.

General

Some civil engineering aspects of atomic power generation

IAN DAVIDSON, *Proceedings*, Institution of Civil Engineers (London), Part I, V. 5, No. 6, Nov. 1956, pp. 703-713

Reviewed by ARON L. MIRSKY

Of primary interest to the structural engineer are the concrete foundation slab, roof slab, and biological shield walls. Economics and structural design of last are stressed.

Paper also discusses, briefly, theory of shielding of graphite-moderated, gas-cooled reactors, erection procedures, and site layout.

Spherical gasometers of concrete (in Dutch)

H.W.F.C. HEMAN, *Cement* (Amsterdam), V. 8, No. 19-20, Aug. 1956, pp. 458-460

Reviewed by JOHN W. T. VAN ERP

Illuminating gas is stored in a completely spherical concrete shell. Above and around the sphere water is stored for displacement of the gas while it is being consumed. Advantages claimed for this system are:

1. Interior of the tank is entirely surrounded by the water storage tank and therefore kept at a nearly constant temperature winter and summer.
2. No moving parts except the water.
3. Low maintenance costs.
4. Good blast resistance, also reasonable bullet proofness of the concrete sphere.
5. Fireproofness.

PERIODICALS REGULARLY SCANNED FOR REVIEWS

The many requests for information concerning the source material of reviews appearing in the "Current Reviews" section indicate the desirabillty of publishing a list of those periodicals, bulletins, and proceedings being *scanned regularly* for articles dealing with the field of concrete. If all miscellaneous magazines, bulletins, reports, and books were included (which are reviewed only as they come to the attention of the voluntary reviewers or editors) the following list would cover well over 200 periodicals and/or publishing, educational, and research organizations. The following list includes publishers' addresses for convenience should correspondence with them be desired, since copies of articles or books reviewed in the JOURNAL are not available through ACI. In most cases they can be obtained directly from the original publishers.

American Ceramic Society Bulletin—American Ceramic Society, 4055 North High St., Columbus 14, Ohio

AREA Bulletin—American Railway Engineering Association, 59 East Van Buren St., Chicago, Ill.

ASTM Bulletin—American Society for Testing Materials, 1916 Race St., Philadelphia 3, Pa.

Annales de L'Institut Technique du Batiment et des Travaux Publics—L'Institut Technique du Batiment et des Travaux Publics, 6 Rue Paul Valéry, Paris 16e, France

Annales des Ponts et Chaussees—Commission des Annales à l'École Nationale des Ponts et Chaussées, 28 Rue des Saints-Pères, Paris 7e, France

Applied Mechanics Reviews—American Society of Mechanical Engineers, 29 West 39th St., New York 18, N. Y.

Architectural Forum—Time & Life Bldg., 9 Rockefeller Plaza, New York 20, N. Y.

Architectural Record—119 West 40th St., New York 18, N. Y.

Australian Journal of Applied Science—Commonwealth Scientific and Industrial Research Organization, 314 Albert St., E. Melbourne, C 2, Victoria, Australia

Bauingenieur, Der—Reichpietschufer 20, Berlin, W 35, Germany

Bauplanung Bautechnik—Unter den Linden 12, Berlin NW 7, Germany

Bauzeitung, Die—Hospitalstrasse 12, Stuttgart N, Germany

Betongen Idag—Norsk Cementforening, Munkedamsveien 3b, Oslo, Norway

Betonstein-Zeitung—Kleine Wilhelmstrasse 7, Wiesbaden, Germany

Beton-Teknik—Christians Brygge 28, Copenhagen V, Denmark

Beton- und Stahlbetonbau—Hohenzollerndamm 169, Berlin-Wilmersdorf, Germany

British Constructional Engineer, The—32 Southwark Bridge Road, London, SE 1, England

Building Research Reports, Danish National Institute of Building Research—Danish National Institute of Building Research, 20 Borgergade, Copenhagen K, Denmark

Building Research Station Digest—Building Research Station, Garston, Watford, Herts, England

Bulletin, Centre d'Etudes et de Recherches de l'Industrie des Liants Hydrauliques—Centre d'Etudes et de Recherches de l'Industrie des Liants Hydrauliques, 197 Boulevard Saint-Germain, Paris 7e, France

Bulletin de la Reunion des Laboratoires d'Essais et de Recherches sur les Materiaux et les Constructions—Reunion des Laboratories d'Essais et de Recherches sur les Materiaux et les Constructions, 12 Rue Brancion, Paris 15e, France

Bulletin of the Faculty of Engineering, Cairo University—Cairo University, Giza, Egypt

Bulletin of the Highway Research Board—Highway Research Board, 2101 Constitution Ave., Washington 25, D. C.

Bulletin of the Institution of Civil Engineers of Ireland—Institution of Civil Engineers of Ireland, 35 Dawson St., Dublin, Ireland

Bulletin del Instituto Tecnico de la Construccion y del Cemento—Instituto Técnico de la Construcción y del Cemento del Patronato Juan de la Cierva de Investigación Técnica, Apartado de Correos No. 2, Costillares, Chamartin, Spain, and Apartado de Correos No. 19,002, Madrid, Spain

Bulletin of the Iowa Engineering Experiment Station—Iowa Engineering Experiment Station, Iowa State College, Ames, Iowa

Bulletin of the University of Illinois Engineering Experiment Station—University of Illinois Engineering Experiment Station, 106 Engineering Hall, Urbana, Ill.

Cahiers du Centre Scientifique et Technique du Batiment—Centre Scientifique et Technique du Batiment, 4 Avenue du Recteur-Poincaré, Paris 16e, France

Cement—Herengracht 507, Amsterdam C, Netherlands

Cement and Lime Manufacture—14 Dartmouth St., London, SW 1, England

Cement och Betong—Kolga-huset, Postgiro 1808, Malmo, Sweden

Cemento, Il—Via Settembrini 9, Milan (503), Italy

Cemento Portland—Calle San Martin 1137, Buenos Aires, Argentina

Ceramic Abstracts—(see *Journal of the American Ceramic Society*)

Civil Engineer Corps Bulletin—Bureau of Yards and Docks, Department of the Navy, Washington 25, D. C.

Civil Engineering—American Society of Civil Engineers, 33 West 39th St., New York 18, N. Y.

Civil Engineering and Public Works Review—8 Buckingham St., London, WC 2, England

Commonwealth Engineer, The—349 Collins St., Melbourne, C 1, Australia

Concrete—400 W. Madison St., Chicago 6, Ill.

Concrete and Constructional Engineering—14 Dartmouth St., London, SW 1, England

Concrete and Reinforced Concrete—State Library of U.S.S.R., Kalinin St. 3, Moscow G-19, Russia

Concrete Construction Magazine—139 N. Clark St., Chicago 2, Ill.

Construction—92 Rue Bonaparte, Paris 6e, France

Constructional Review—14 Spring St., Sydney, N.S.W., Australia

Consulting Engineer—227 Wayne St., St. Joseph, Mich.

Contractors and Engineers—470 Fourth Ave., New York 16, N. Y.

Crushed Stone Journal, The—National Crushed Stone Association, 1415 Elliot Place, NW, Washington 7, D. C.

Engineer, The—28 Essex St., Strand, London, WC 2, England

Engineering—35 & 36 Bedford St., Strand, London, WC 2, England

Engineering Journal, The—2050 Mansfield St., Montreal 2, Quebec, Canada

Engineering News-Record—330 West 42nd St., New York 18, N. Y.

Genie Civil, Le—5 Rue Jules Lefebvre, Paris 9e, France

Geotechnique—Institution of Civil Engineers, Great George St., London, SW 1, England

Giornale del Genio Civile—Via Nomentana 2, Rome, Italy

Gradbeni Vestnik—Erjavceva 11a, Ljubljana, Yugoslavia

Handlingar (Proceedings), *Swedish Cement and Concrete Research Institute*—Svenska Forskningsinstitutet for Cement och Betong vid Kungl. Tekniska Hogskolan I, Stockholm, Sweden

Highway Research Abstracts—Highway Research Board, 2101 Constitution Ave., Washington 25, D. C.

Housing Research Papers, HHFA—Housing and Home Finance Agency, Division of Housing Research, Washington 25, D. C.

Hungarian Technical Abstracts—Hungarian State Technical Library, P. O. Box 12, Budapest 8, Hungary

Indian Builder, The—Ionic, off Arthur Bunder Rd., Colaba, Bombay 5, India

Indian Concrete Journal, The—Concrete Association of India, 121 Queen's Road, Bombay, India

Industria Constructiilor si a Materialelor de Constructii—Revistele Tehnice ASIT, Str. loan Ghica nr. 3, Bucharest, Rumania

Industrial and Engineering Chemistry—American Chemical Society, 1155 Sixteenth St., NW, Washington 6, D. C.

Informes de la Construccion—Institute Técnico de la Construcción y del Cemento del Patronato Juan de la Cierva de Investigacción Tecnica, Apartado de Correos No. 2, Costillares, Chamartín, Spain, and Apartado de Correos No. 19,002, Madrid, Spain

Ingenieria Civil—Plaza de la Republica No. 55-602, Apartado Postal 10423, Mexico 1, D. F., Mexico

Ingenieria Internacional Construccion—330 West 42nd St., New York 36, N. Y.

Ingeniøren—Dansk Ingeniørforening, Ingeniorhuset, Vester Farimagsgade 29, Copenhagen V, Denmark

Journal of the Boston Society of Civil Engineers—Boston Society of Civil Engineers, 715 Tremont Temple, Boston, Mass.

Journal of the American Ceramic Society and Ceramic Abstracts—American Ceramic Society, 4055 N. High St., Columbus 14, Ohio

Journal of the Institution of Engineers, Australia—Institution of Engineers, Gloucester & Essex Sts., Sydney, N. S. W., Australia

Journal of the Japan Society of Civil Engineers—Japan Society of Civil Engineers, No. 4, 2-Chome, Ohte-Machi, Chiyoda-Ku, Tokyo, Japan

Journal of Research—National Bureau of Standards, Washington 25, D. C.

Magazine of Concrete Research—Cement and Concrete Association, 52 Grosvenor Gardens, London, SW 1, England

Meddellanden (Bulletins), *Statens Provningsanstalt*—Statens Provningsanstalt, Stockholm, Sweden

Memoires de la Societe des Ingenieurs Civils de France—Society of Civil Engineers of France, Hotel de la Societe, 19 Rue Blanche, Paris 9, France

Military Engineer, The—Society of American Military Engineers, Mills Bldg., Pennsylvania Ave. at 17th St., NW, Washington 6, D. C.

Nase Gradevinarstvo—Association of Societies of Civil Engineers and Technicians of Yugoslavia, Kneza Milosa 7, Belgrade, Yugoslavia

Nordisk Betong—Nordisk Betonforbund (Nordic Concrete Association), Drottning Kristinas väg 26, Stockholm, Sweden

Oesterreichische Bauzeitschrift—Mölkerbastei 5, Vienna I, Austria

Ontario Hydro Research News—Research Division, Hydro-Electric Power Commission of Ontario, 620 University Ave., Toronto 2, Ont., Canada

Pit and Quarry—431 S. Dearborn St., Chicago 5, Ill.

Polish Technical Abstracts—Centralny Instytut Dokumentacji Naukowo-Technicznej, Aleja Niepodleglosci 188, Warsaw, Poland

Polytechnisch Tijdschrift—28 Balistraat, The Hague, Netherlands

Precontrainte—21 Rue Newton, Brussels 4, Belgium

Prefabrication—32 Millbank, London, SW 1, England

Proceedings of the American Society of Civil Engineers—American Society of Civil Engineers, 33 West 39th St., New York 18, N. Y.

Proceedings of the American Society for Testing Materials—American Society for Testing Materials, 1916 Race St., Philadelphia 3, Pa.

Proceedings of the Deutscher Ausschuss fur Stahlbeton—Bundesallee 215/218, Room 302, Berlin, W 15, Germany

Proceedings of the Deutscher Beton-Verein—Deutscher Beton-Verein E. V., Bahnhofstrasse 61, Postfach 627, Wiesbaden, Germany

Proceedings of the Highway Research Board—Highway Research Board, 2101 Constitution Ave., Washington 25, D. C.

Proceedings of the Institution of Civil Engineers—Institution of Civil Engineers, Great George St., Westminster, London, SW 1, England

Publicaciones Tecnicas del Instituto del Cemento Portland Argentino—Instituto del Cemento Portland Argentino, Calle San Martin 1137, Buenos Aires, Argentina

Publications of the International Association for Bridge and Structural Engineering—Stockerstrasse 64, Zurich 39, Switzerland

Public Roads—Bureau of Public Roads, U. S. Department of Commerce, Washington 25, D. C.

Rakennusinsinoori—Ratak 9, Helsinski, Finland

Reinforced Concrete Review, The—Reinforced Concrete Association, 94-98 Petty France, London, SW 1, England

Revista de Engenharia Mackenzie—Rua Maria Antonia 403, Sao Paulo, Brazil

Revista del Colegio de Ingenieros de Venezuela—Parque Los Caobos, Apartado 2.006, Caracas, Venezuela

Revue des Materiaux de Construction—Centre d'Etudes et de Recherches de l'Industrie des Liants Hydrauliques, 19 Rue Lafayette, Paris 9e, France

Roads and Streets—24 W. Maple St., Chicago 10, Ill.

Road Research Notes—Road Research Laboratory, Harmondsworth, Middlesex, England

Rock Products—79 W. Monroe St., Chicago 3, Ill.

Schweizerische Bauzeitung—Schweizerischer Ingenieur-und Architekten-Verein, Postfach, Zurich 39, Switzerland

Structural Engineer, The—Institution of Structural Engineers, 11 Upper Belgrave St., London, SW 1, England

Technical Memorandum, Waterways Experiment Station—Waterways Experiment Station, Corps of Engineers, Vicksburg, Miss.

Tecnica—Revista de la Facultad de Ciencias Exactas y Tecnologia, Universidad Nacional de Tucuman, Argentina

Tekniak Ukeblad—Oslo, Norway

Transactions of the Institution of Civil Engineers of Ireland—Institution of Civil Engineers of Ireland, 35 Dawson St., Dublin, Ireland

Transactions of the Japan Society of Civil Engineers—Japan Society of Civil Engineers, No. 4, 2-Chome, Ohte-Machi, Chiyoda-Ku, Tokyo, Japan

Travaux—6 Avenue Pierre 1er-de-Serbie, Paris 16e, France

VDI Zeitschrift—Verein Deutscher Ingenieure-Verlag GmbH, Ingenieurhaus, Prinz-Georg-Strasse 77, Düsseldorf, Germany

World Construction—24 W. Maple St., Chicago 10, Ill.

Zement-Kalk-Gips—Kleine Wilhelmstrasse 7, Wiesbaden 16, Germany

Zement und Beton—Oesterreichischen Betonvereins und Verein der Oesterreichischen Zementfabrikanten, Strohgasse No. 21 A, Vienna 3, Austria

Discussion of a paper by Thomas Cholnoky:

Prestressed Concrete Pavement for Airfields*

By L. A. PALMER†

The study of possibilities for economical design by prestressing concrete pavements is timely. The excellent report by Mr. Cholnoky is a clear and brief description of the progress of this study. The use of concrete in certain critical areas of Navy airfields is a "must." Critical areas include all paved sections of runways, taxiways, warm-up aprons, and parking areas that are exposed to the hot blasts of jet aircraft. Heat and blast create the only serious problem. Fuel spillage tends to be a problem of minor importance and may be solved to a great extent by "good housekeeping."

The prestress investigation is a part of the general study sponsored and conducted by the Bureau of Yards and Docks in its search for suitable and economical materials and design procedures that can meet the new requirements of planes of the future. The likelihood that future designs of military aircraft may punish pavements to a far greater extent than those now in operation is sufficient to warrant this research. The general investigation by this Bureau includes a search for concrete aggregates which will impart to the concrete such properties as low thermal expansivity and low thermal conduction without serious sacrifice of strength in flexure. In this, considerable progress has been made and it is indicated that these desirable properties can be realized with those types of portland cement now normally used in the construction of concrete pavement not subjected to jet plane traffic.

One main consideration in this general study is the cost of immobilizing a runway or possibly even an entire airfield to repair the damage to the pavement by modern military aircraft. The interruption of training schedules, incident to such maintenance, tends to be much more costly to the nation than the relatively minor cost of materials and labor. In time of war the cost of such immobilization would be inestimable. This fact must be appreciated and thoroughly borne in mind.

It should be possible to eliminate all joints, except construction joints, by post-tensioning or prestressing the concrete pavements in the critical areas. The elimination of joints to this extent would indeed be a great advance. Where jets operate, a joint is the "Achilles heel" in concrete pavement. The actual cost of prestressing diminishes in importance as a factor when one con-

*ACI Journal, July 1956, *Proc.* V. 53, p. 59. Disc. 53-3 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Engineering Consultant, Soil Mechanics and Paving, Bureau of Yards and Docks, Department of the Navy, Washington, D. C.

siders the cost of time out for repairs of a "hot" runway. Such a runway tends to be a potential modern military weapon and is like the deck of a carrier.

Cracks are as vulnerable as joints unless they are restrained from opening. With well selected coarse aggregate and with the slab held in compression by prestressing there is almost perfect load transfer and no need to seal the cracks. Such cracks relieve tension in the lower fiber of the slab when there is warping due to heating of the upper surface.

The reduction in slab thickness made possible by prestressing tends to greater flexibility and thereby less vulnerability to fatigue due to load repetition. Flexibility and continuity and permanency of adequate subbase support should go far toward reducing the effect of fatigue. A good subbase is not a luxury. There is no procedure, either rational or empirical, for designing concrete pavement which must have "mushroom" support. Furthermore, the friction coefficient of slab on subbase should be controllable within certain limits. Its reduction should be made a separate subject for research and the results of such a study may be tied in with the economics of prestressing.

The load capacity of the 7 in. thick post-tensioned slab described by the author has exceeded all expectations. According to Westergaard theory the slab should have failed in bending under the 200-kip load; in fact, it should have failed at a much lesser load. The data indicate that the measured deflections are in agreement with those computed from the Westergaard theory but the curvatures of the deflection curves definitely are not. Since the Westergaard theory assumes no condition of prestressing it would seem wrong to consider that the compression under prestressing is equivalent to added strength in flexure. This writer suggests that in a rational analysis, the approach might be that the prestressing should be considered as applied external load. The condition is similar to that existing in a triaxial test whereby the shearing stress of a cylindrical sample of clay or of stone is greatly increased by a chamber (lateral) pressure, normal to the applied axial load.

Discussion of a paper by S. J. Chamberlin:

Spacing of Reinforcement in Beams*

By PHIL M. FERGUSON and J. NEILS THOMPSON†

Professor Chamberlin's careful study is not only significant for what it tells about bar spacing but also because of what it implies about the inadequacy of present bond specifications. The data clearly indicate that few reinforcing bars in actual beams are so placed that their adequacy in bond can be safely evaluated on the basis of the present ACI Building Code.

This discussion attempts: (1) to evaluate the reported bond values in terms of the ACI Building Code allowable and to point out the weakness in the present allowable; (2) to indicate the need for further research on bond; (3) to point out how some of these data agree with results from eccentric pullout specimens and beams already reported; and (4) to explain the probable reason for differences between these results and those of the "shallow bead" specimens.

Professor Chamberlin reports 53 lines of data in Table 3, many of these lines representing average values for three companion specimens. This was a careful investigation, with a broad range of variables, different embedments, different bar spacings, and different concrete strengths.

For a discussion of bond strength of modern deformed bars, 22 lines of data must be excluded:

Four lines of Series II devoted to plain bars.
Five lines of Series I where old-type deformed bars are reported.
Five lines in Series III and IV representing tension failures.
Eight lines of Series III representing bar embedments of only 3-in.

Although the 3-in. embedment is important in completing the research picture it does not represent a practical condition. There remain 31 lines of pertinent data which may logically be compared with the allowable bond stress of 350 psi permitted by the ACI Building Code for a concrete strength of 3500 psi or more.

Against a failure as troublesome and as tricky as bond failure, a factor of safety of 2.5 seems most desirable. Only two lines of data for higher strength concrete meet this test satisfactorily (and one of the tension failure lines possibly should also be included) compared to 29 lines showing a lower factor of safety:

*ACI Journal, July 1956, *Proc.* V. 53, p. 113. Disc. 53-6 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Chairman, Department of Civil Engineering, and Professor of Civil Engineering, respectively, University of Texas, Austin, Tex.

Factor of Safety	
Less than 2.5	More than 2.5
4 for 3680-psi concrete 22 for 4500-psi concrete 5 for 5870-psi concrete	0 for 3680-psi concrete 1 for 4470-psi concrete 1 for 5870-psi concrete

Some engineers might be satisfied with a factor of safety of 2.0, but 25 of these 31 lines show data not meeting this requirement, including: 4 for 3680-psi concrete; 18 for 4500-psi concrete; and 3 for 5870-psi concrete.

It is hoped that no one would accept a factor of safety for bond as low as 1.5, but half of these 31 lines of data fail to provide even this skimpy factor: 2 for 3680-psi concrete; 13 for 4500-psi concrete; and 1 for 5870-psi concrete.

It must be concluded that Professor Chamberlin's data provide no justification for a 350-psi working stress for bond. Are his data valid in this connection? Some might argue that a ½-in. clear bar spacing is prohibited under the Code, as it is. However, a 1-in. clear spacing would be permitted with a #8 bar, which gives the same one-diameter clear spacing. Those who have made bond tests on bars will probably agree that, if any difference exists in bond action of a #4 bar as compared to a #8 bar, the #4 bar will perform more favorably. To these discussors Professor Chamberlin's tests appear to be entirely valid, although his bond values appear somewhat high for normal conditions, as noted later.

As pointed out elsewhere,[1] bond and diagonal tension strength are closely related. If this were not so, these low bond strengths would be alarming. Under our present Code, where the present high allowable bond stress might seem to permit too small a beam, the allowable shear stress usually controls the beam size. The shear stress thus requires a beam large enough to reduce the actual bond stress to a value more in line with these test values.

These discussors are entirely in agreement with Professor Chamberlin's statement that bond failure of deformed bars in a beam is not a matter of splitting alone but a combination of shear and splitting. It is entirely possible that in some cases the shear (diagonal tension) may be the critical element with the splitting more of an aggravating mechanism than a primary cause. Certainly the correlations between bond stress, bar spacing, and shear strength which were reported earlier[1] seem to be too definite to be accidental.

More research is clearly justified and required to put bond specifications on a sound basis. Such a basis is impossible unless a clear distinction can be drawn between (1) what might be called the potential bond strength as measured in confined concrete and (2) the bond strength in structural members which are subject to complications of splitting and diagonal tension. Unfortunately the major research to date has been on bond strength in confined concrete and such is the chief basis of the present ACI Code. The result is an allowable bond stress which is safe in mass concrete and in spiral columns and probably in some other undefined situations; but this bond stress is definitely unsafe

for splices.[2] Professor Chamberlin's tests seem to indicate that lower allowable bond stresses are also necessary to protect the safety of most structural beams. This conclusion might also be drawn from tests reported[1] from the University of Texas.

It is pleasing to these discussors to note that on the comparable "deep bead" specimens Professor Chamberlin's results check closely with the University of Texas data. Only six of these deep bead specimens are reported. These six values of bond have been plotted in Fig. A alongside the University of Texas curve. Considering the difference in specimens, the agreement is unexpectedly close; only one of the six points is significantly different. Why the 24-diameter embedment gives bond values larger than a 12-diameter embedment is not clear, but the difference is small. The University of Texas curve is based on a 12-diameter embedment.

In the author's Fig. 17, part of this same curve is shown as a dotted line and values from the author's shallow bead specimens are plotted for comparison. In general the author's values are higher for a given bar spacing, but the order of magnitude is the same. Either curve would adequately support the conclusion that lower allowable bond stresses are needed for general usage.

To these discussors the lower dotted curve in Fig. 17, which they originated, still appears to be the more representative of actual beam conditions, for the reasons which follow. Their curve originated from sections more like those

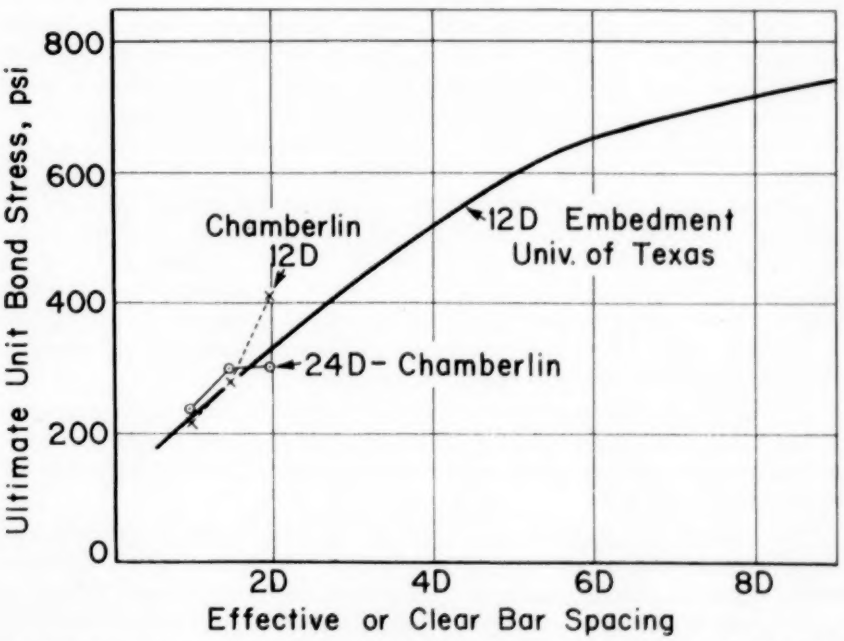


Fig. A—Comparison of Chamberlin's results for "deep bead" beams with University of Texas eccentric pullout curve

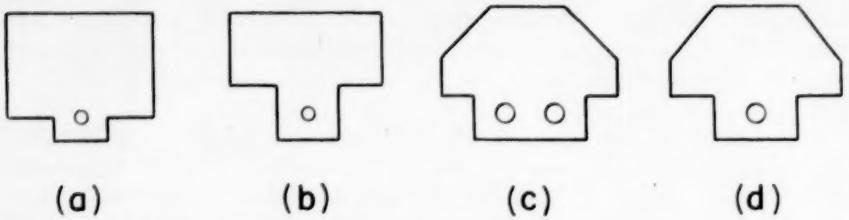


Fig. B—(a) Chamberlin's "shallow bead" beam; (b) Chamberlin's "deep bead" beam; (c) and (d) University of Texas eccentric pullout specimens

designated by Professor Chamberlin as deep bead beams as shown in Fig. B(b), that is, the bars were always placed entirely clear of any enlarged section, as shown in Fig. B(c) and (d). In the author's shallow bead specimens the bar was centered at the face of the enlarged section as shown in Fig. B(a). Since in a typical beam the critical or weakest section lies above the middle of the bar, the shallow bead specimen of Fig. B(a) is reinforced in width at this critical section. Possibly this statement needs some further explanation.

It is assumed that failure results from a combination of splitting due to bond and of horizontal shear on the concrete between the bars. The splitting stress is maximum on the axis of the bar, but it is large anywhere in that vicinity. Horizontal shear is also large on this minimum section between the bars but the maximum unit shear occurs substantially above that plane. For a clear spacing of one bar diameter, D, the maximum unit shear plane lies approximately $D/4$ above the minimum throat section and this unit shear is about 18 percent larger than at the minimum section. For a clear spacing of $2D$, the maximum shear is just below the top of bar and is about 33 percent larger. For a clear spacing of $3D$, the maximum shear is at the top of bar and 50 percent larger. It thus seems certain that the maximum tension due to combined splitting and shear typically lies well above the minimum throat section at a plane where the entire width of the shallow bead specimens of Fig. B(a) provides added resistance. Hence such a specimen will be stronger than the deep bead specimen which has no such extra width. In the opinion of the discussors the shallow bead specimen is also stronger than practical beams which do not have the protective extra width.

Bond strength in lapped splices has had more study.[2] Since splitting at splices is not much aggravated by horizontal shear, larger bond stresses are permissible; but even in splices, safe stresses are probably only about half the present Code allowable.

The fact that this discussion calls attention to need for a change in the ACI Building Code should not be interpreted as a criticism of ACI Committee 318.* The committee cannot yet set proper bond values with certainty, pending still further needed research, and such research is not even underway insofar as is now known. But Professor Chamberlin's data do provide enough

*One of the discussors is a member of this committee.

confirmation of weakness in bond to demand specification recognition in the form of lower allowable bond stresses in unconfined structural concrete. More conservative values are not only essential where weakness has been established but such values should also be used wherever there is any question as to strength.

REFERENCES

1. Ferguson, Phil M., Turpin, Robert D., and Thompson, J. Neils, "Minimum Bar Spacing as a Function of Bond and Shear Strength," ACI JOURNAL, June 1954, *Proc.* V. 50, pp. 869-887; discussion: pp. 888.1-888.8.

2. Chinn, James, Ferguson, Phil M., and Thompson, J. Neils, "Lapped Splices in Reinforced Concrete Beams," ACI JOURNAL, Oct. 1955, *Proc.* V. 52, pp. 201-213.

Discussion of a paper by Phil M. Ferguson:

Some Implications of Recent Diagonal Tension Tests*

By JOSEPH TAUB and AUTHOR

By JOSEPH TAUB†

Professor Ferguson's excellent paper represents a significant contribution to the exploration of the behavior of reinforced concrete beams failing in shear. The problem involves so many factors that it merits further discussion.

In the writer's opinion the shear failure of a reinforced concrete beam manifests itself in either of two ways: (a) as a shear-compression failure, or (b) as a shear-tension failure. Both cases can take place, of course, simultaneously.

The term "shear-compression failure" designates that the primary failure cause is the inadequate shear resistance of the beam, development of a diagonal tension crack, which is followed by destruction of the compression zone above the top end of this crack.

The term "shear-tension failure" designates that the primary failure cause is as described above, which is followed by destruction of the tension zone between the bottom end of the diagonal crack and the support of the beam.

Both cases are discussed here.

SHEAR-COMPRESSION FAILURE

This kind of failure occurs in rectangular beams only, as their compression zone is relatively small. The writer agrees with the author that this kind of failure takes place in beams where the a/d value is relatively small.

As the diagonal tension crack increases it moves upward, toward the top surface of the beam; the compression zone of the beam becomes reduced, and this causes destruction of the concrete above the top end of the crack. A typical example of a shear-compression failure is represented by Beam III-25a‡ in the tests reported by Moody, Viest, Elstner, and Hognestad.[1]

The ultimate strength of a beam failing in shear-compression depends on the arrangement of the loads, on the magnitude of the stress in concrete in the compression region above the diagonal crack, and on the concrete strength f_c'. This stress is a function of (1) the bending moment and shear at the critical section, (2) the proportion of the beam, (3) the percentage of the reinforcement, (4) the distance of the neutral axis from the top surface of the

*ACI Journal, Aug. 1956, *Proc.* V. 53, p. 157. Disc. 53-8 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Head of Structural Department, Workers Housing Co., Ltd., Tel-Aviv, Israel.

‡See Fig. 4 in Reference 1.

beam, and (5) the shape of the stress block at ultimate load. A formula for the ultimate strength of such beams must take into account all these factors.

In the case of a shear-compression failure the concrete strength plays a significant role, as may be seen from the tests by Clark[2] and Moody.[3]

The tension in the longitudinal reinforcement at the bottom end of the diagonal crack is not critical, and the effect of this reinforcement manifests itself as follows: the higher percentage of tension steel increases the moment of inertia of the beam, reduces the opening of the diagonal crack, and so its destructive action. The tests reported by Moody, Viest, Elstner, and Hognestad[1] showed that with increasing percentage of tension steel the midspan deflection of the beam decreased. This smaller beam deflection proves also that the width of the cracks was smaller.

SHEAR-TENSION FAILURE

This kind of failure occurs in rectangular beams with relatively great a/d. As the diagonal tension crack crosses the tension steel a redistribution of internal forces takes place, the tensile force in the steel at the bottom end of diagonal crack increases greatly, and this leads to failure of the beam.

It is irrelevant whether the diagonal crack starts at middepth or at bottom of the beam; it never opens horizontally but always in a diagonal direction. At the intersection with the tension steel it follows this steel in the direction of its opening; the force in the steel at this point, therefore, acts diagonally. The vertical component of this force, and the horizontal as well, cause splitting of the beam along the tension steel, beginning from the bottom end of the diagonal crack up to the beam support. In this way the bond between tension steel and concrete is completely destroyed, and this causes beam failure.

The intensity of this action depends on the magnitude of internal force in the tension steel at the bottom end of the diagonal crack.

Fig. A shows two beams failing in shear. Fig. A(a) corresponds to Fig. 4 and shows a beam in which a/d is relatively great. Fig. A(b) corresponds to Fig. 2 and shows a beam with small a/d. The internal force in tension steel at the bottom end of the diagonal crack depends on the magnitude of bending moment at its top end. Hence it follows, that at equal loads this force is much greater in the beam represented by Fig. A(a) than in that shown by Fig. A(b). The internal force in the compression zone of the first beam is, of course, greater too. Comparing the crack pattern of both these two beams, it is seen that

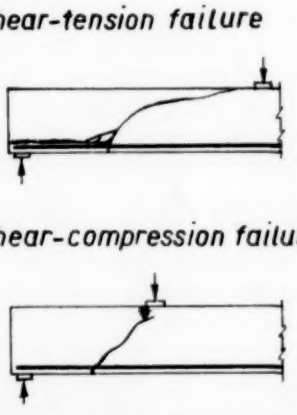


Fig. A—Crack pattern for shear-tension (a) and shear-compression (b) failure

the upper part of the diagonal crack in the first beam has a flat slope. Its compressive zone is, therefore, large enough to carry the force that acts on it. This explains the shear-tension failure of beams with relatively great a/d.

A typical example of a rectangular beam failing in shear-tension is represented in Fig. 3. Typical examples of simultaneous occurrence of shear-compression and shear-tension failure of rectangular beams are the beams of Series B* in the tests reported by Moody, Viest, Elstner, and Hognestad.[1]

As some German tests showed, in T-beams with their relatively large compression zone, the shear-tension failure always takes place. Table A lists some tests on T-beams by Bach and Graf.[4]

From Table A it is seen that in beams which were provided with vertical stirrups no splitting of the beam occurred along the tension steel. The effect of hooks at the tension steel ends in these beams was significant.

Some tests reported by J. Neils Thompson and the author will also be discussed. From the tests on T-beams of Joistile type[5] two test beams of Series A are selected: H-13 and H-3.

The beams were identical except that tension steel in Beam H-13 was welded to an end anchorage plate and Beam H-3 had no anchorage. In both beams no web reinforcement was provided. Concrete strength of the two beams was nearly equal. Span of the beams was 84-in. The beams were loaded by two loads at 1/3-points. The a/d ratio was 6.2.

The nominal shear stress at failure was 345 psi for Beam H-13 and 260 psi for Beam H-3. End anchorage of the tension steel in Beam H-13 increased the ultimate strength of this beam by 33 percent.

From later tests on T-beams reported by the author and J. Neils Thompson[6] the beams in Series C will be discussed. The beams were provided with "shoulders" to examine their effort on the ultimate strength.

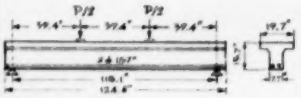
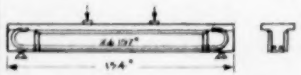
Span of the beams was 64-in. The beams were loaded by two equal loads at 28-in. from the support. The a/d ratio was 3.39. Two equal beams were tested. In both the tension steel was welded to an end anchorage steel block. No web reinforcement was provided. The ultimate strength of Beam C1 was 22.54 kips, that of Beam C2 was 17.44 kips.

In the authors' opinion the higher ultimate strength of Beam C1 seemed to be a matter of tied arch action. The writer agrees with this opinion. But, after formation of a diagonal crack and redistribution of internal forces the tied arch action is a normal phenomenon of diagonal tension failure in beams in which the tension steel is anchored at its ends—be it by anchorage plates or by hooks. Fig. 5 of the respective paper,[6] illustrating the beam at failure, shows a large diagonal crack, splitting of the beam along the tension steel from the bottom end of this crack up to the beam support, and destruction of the beam end by the anchorage block.

The beams of Group 7 in the German tests, included in Table A, in which hooks were provided at the tension steel ends, showed exactly the same crack pattern at failure.

*See Fig. 5 in Reference 1.

TABLE A—RESULTS OF BEAM TESTS BY BACH AND GRAF

Group	Type of beam*	P_s† kips	P_u kips	Increase in ultimate strength by providing hooks at the tension steel end. percent	Failure cause
1		35.3	36.0	—	Formation of a diagonal crack which causes bending down of the tension steel at the bottom end of the crack; splitting the beam along the longitudinal reinforcement from the diagonal crack up to the beam support.
7		48.5	54.2	52	As above followed by the destruction of beam end by the hooks of tension steel.
4		44.1	48.5	—	Formation of a diagonal crack slipping of the tension steel.
12		59.5	79.5	64	As above followed by the destruction of beam end by the hooks of tension steel.

* The beams were reinforced with plain round bars.

† The load under which first slipping at the tension steel end was observed.

The difference between the ultimate strength of Beam C1 and C2 may be considered as scatter of test results.

From the above remarks the following conclusions can be drawn:

1. Bond failure is never a primary failure cause. It shows that the shear resistance of the beam is inadequate.
2. The stirrups, besides their role as web reinforcement, fulfill a second task in reinforced concrete beams by hindering the splitting of the beam along the tension steel between the bottom end of the diagonal tension crack and the beam support.
3. The use of deformed instead of plain bars is more effective in beams which are provided with stirrups than in those without web reinforcement.
4. In beams in which a shear-tension failure comes into question, the hooks at the ends of tension steel play a significant role. The ultimate strength of such beams depends to a great degree on the pressure exerted by the hooks on concrete and on the resistance of the concrete against this pressure.
5. In beams which fail in shear-tension, the compressive force above the diagonal crack is not critical. Higher concrete strength is shown by delayed formation of initial cracks and greater resistance to action of tension steel and hooks, if any.

The shear strength of a beam depends on all the above mentioned factors. The exploratory tests of the author showed that there are still many other factors which affect it to a great degree. It would be difficult indeed to find formulas for predicting the ultimate shear strength of reinforced concrete beams which would include all these factors.

In the writer's opinion, however, it should not be the aim to find such formulas; the aim should be to provide the beam with such web reinforcement that no shear failure may occur. The tests by Moody[3] showed that under certain circumstances a shear failure can occur simultaneously with the formation of the first diagonal crack. In the case of a moment failure the progressive cracking and deflection of the beam warn us of eventually nearing danger.

EFFECT OF MULTIPLE LOADS

The exploratory tests by the author of Beams F2, F3, and F5 will now be discussed.

The dimensions of all these beams were equal; their span was 60-in. Beam F2 was loaded with two symmetrical loads 24-in. from the support. In Beams F3 and F5 two additional symmetrical loads were placed within the 24-in. shear span. All the beams failed in diagonal tension. The failure load of Beams F3 and F5 was about 60 percent higher than that of Beam F2.

Fig. 12 shows diagrammatically the crack pattern of the beams at failure. It was nearly identical for all these beams. The various kinds of loading were of no influence on the crack pattern. The first diagonal crack occurred in Beam F2 at an average shear of 3750 lb, in Beam F3 at an average shear of 5500 lb, and in Beam F5 at an average shear of 5000 lb.

The author attributes the higher value of shear at first diagonal crack in Beam F3 and F5 to the action of exterior loads on these beams. In the writer's opinion still another factor played a dominant role.

As was already mentioned before, there is a close connection between the deflection of a beam and the crack development in it. The smaller the beam deflection, the later the first crack forms and the slower also is its progress, and vice versa. The tests by Moody[3] showed that the deflection of T-beams were much smaller than those of corresponding rectangular beams. The first diagonal cracks in T-beams also formed at higher loads than in rectangular beams. The deflection of a beam depends not only on its moment of inertia but also on the shape of the bending moment curve.

In the author's paper, load-deflection curves of the beams were not shown. But the computed deflections of Beam F2 are much greater than those of Beams F3 and F5. This explains the delayed formation of the first diagonal cracks in these last beams.

As it is seen from Fig. 12, all the beams failed in shear-tension. Failure was caused by the action of the internal forces in the tension steel at the bottom end of the diagonal crack. It is known that these forces depend on the magnitude of the bending moment at the top end of the crack. The writer

gathers from Fig. 12 that the top end of the diagonal crack was: in Beam F2 at a distance $a = 24$ in. from the support, in Beam F3 at a distance $a = 23$ in. from the support, and in Beam F5 at a distance $a = 21$ in. For these values of a the bending moments at the top end of the diagonal crack under the failure load, yield for Beam F2: $M = 120$ in.-kips, for Beam F3: $M = 139$ in.-kips, and for Beam F5: $M = 135$ in.-kips. The differences between these values are not great. Although the failure load of Beams F3 and F5 was much greater than that of Beam F2, in all these beams the internal force in the tension steel at the bottom end of the diagonal crack, which was the immediate cause of failure, was nearly equal.

The smaller value of the bending moment at the top end of the diagonal crack for Beam F2 or, as it may be said, the somewhat earlier failure of this beam, resulted from its greater deflection which caused an earlier formation of the diagonal crack and its more rapid progress.

It seems probable that investigators would appreciate the inclusion of load-deflection curves and data on the progress of cracks for the respective beams in the author's closure.

From the above tests it is seen that the normal formula for computation of the shearing unit stress $v = V/bjd$ cannot be considered as a shear strength measure of reinforced concrete flexural members. This formula is based on the elastic theory which does not apply to the ultimate strength of such members. There is an urgent need of further research in this field to get a clear view of the whole problem of shear resistance of reinforced concrete beams.

REFERENCES

1. Moody, K. G., Viest, I. M., Elstner, R. C., and Hognestad, E., "Shear Strength of Reinforced Concrete Beams. Part 1—Tests of Simple Beams," ACI JOURNAL, Dec. 1954, *Proc.* V. 51, pp. 317-332.

2. Clark, A. P., "Diagonal Tension in Reinforced Concrete Beams," ACI JOURNAL, Oct. 1951, *Proc.* V. 48, pp. 145-156.

3. Moody, K. G., "An Investigation of Reinforced Concrete Beams Failing in Shear," PhD Thesis, Department of Theoretical and Applied Mechanics, University of Illinois, June 1953.

4. Bach, C., and Graf, O., "Versuche mit Eisenbeton-Balken zur Ermittlung der Widerstandsfachigkeit verschiedener Bewehrung gegen Schubkraefte. Erster Teil," *Heft* 10, Deutscher Ausschuss fuer Eisenbeton, Berlin, 1911.

5. Thompson, J. N., and Ferguson, P. M., "Shear Resistance of Tile-Concrete Floor Joists," ACI JOURNAL, Nov. 1950, *Proc.* V. 47, pp. 229-236.

6. Ferguson, P. M., and Thompson, J. N., "Diagonal Tension in T-Beams Without Stirrups," ACI JOURNAL, Mar. 1953, *Proc.* V. 49, pp. 665-676.

AUTHOR'S CLOSURE

In the 20 months since this paper was first prepared many additional diagonal tension tests have been made on beams without web reinforcement, exploring various aspects of this intriguing problem. It seems desirable to give added information on several conditions specifically mentioned in the paper.

The first of these relates to the effect of percentage of longitudinal steel upon the diagonal tension strength of a beam. The paper suggested that this should have only a small effect on shear strength. In the case of T-beams, with an a/d of approximately 4, doubling the steel area has resulted in practically no change in shear resistance. On the other hand, such seems not to be true with rectangular beams. An increase in steel area increases the depth to the neutral axis and keeps moment cracks small. The resulting increased area above cracks is quite noticeable, even after cracks become somewhat inclined. An increased resistance to shear failure does accompany this extra steel condition in rectangular beams. Why T-beams and rectangular beams seem to respond differently to variations in percentage of steel is not clear to the author. It must be noted that these observations are based on only a few pilot tests; further investigation seems to be indicated.

Related to percentage of steel is the question of how beams behave when over-reinforced to such an extent that a moment failure would have to be a failure in compression. The hypothesis suggested that a diagonal tension crack might trigger a premature moment failure in such a case. A few rectangular beams heavily over-reinforced for moment and test loaded with a single concentrated load at midspan did fail suddenly by what appeared to be a diagonal tension failure, with an "unrestrained" type of diagonal crack. However, analysis of the data indicated that the shear resistance exceeded that of less heavily reinforced beams while the moment in three out of four beams exceeded $0.40\, f_c' bd^2$. Although the type of failure indicated shear involvement, there was certainly no apparent adverse effect on strength. The effect of increased steel percentage appeared to offset the ill effects expected from critical compression stresses.

Another situation briefly mentioned in the paper was that existing at a point of inflection in a continuous beam. The analysis suggested that a diagonal tension crack near a point of inflection, once started, would proceed quickly to complete failure. However, as also indicated in the paper, such diagonal cracks seem to start more readily at points where moment exist. Such failures from shear, starting near the point of inflection, tend to occur suddenly and violently unless stirrups are present. Negative moment steel splits off, apparently the result of combined dowel action and bond stress. It further appears that diagonal tension cracks will not initially form *at* the point of inflection itself until shears are increased possibly 40 percent above those causing the failure in the presence of moment.

In rectangular beams, diagonal tension cracks starting near middepth appear to be restricted to beams with small a/d ratios. Certainly the continuous beam and the simple beam with large a/d ratio indicate that diagonal cracks develop more readily from moment cracks.

Dr. Taub has presented a thoughtful discussion of the paper. While the author agrees with his remarks for the most part, some portions require comment. The shear-tension failure is a case in point. In discussing this type of failure, Dr. Taub seems to center his attention upon the tension steel

while the author would subdivide this into two cases: (1) failure along the flat slope in the compression zone, and (2) failure on the tension side. The tension failure might always occur with plain bars; the author has had little experience with plain bars. However, with deformed bars, many beams fail in the compression zone with the end of the tension bars still intactly embedded in the concrete. In beams with shorter a/d ratios, welded anchor plates increase tensile resistance but horizontal splitting can finally proceed over the top of such plates.

The unrestrained failure in the compression zone is considered by the author as the typical basic failure, with the tension failure considered as a secondary effect. This may be open to discussion. Certainly the so-called dowel action which tends to split off the tension steel appears to be accompanied by a similar tension tending to split off the compression slab of concrete. The combination of the ordinary shear and compression on this "slab" at the top of the beam with some direct tension perpendicular to and just beyond the crack seems to give a tension splitting type of failure. The author does not regard this type of crack, shown at the top of Fig. A, as basically a shear-compression failure.

The author must disagree vigorously with Dr. Taub's first conclusion, unless it is greatly restricted: "Bond failure is never a primary failure case. It shows that the shear resistance of the beam is inadequate."

In contrast, the author would state that the slip of bars caused by inadequate bond resistance can and usually will trigger a *premature* diagonal tension failure. Likewise, as Dr. Taub points out, a diagonal tension crack will often lead to a premature failure in bond. Bond and diagonal tension seem to be inseparable in action. In many cases only the sequence of events can establish primary and secondary cause; and this sequence at times is extremely rapid.

In Dr. Taub's second conclusion relating to the role of stirrups in hindering splitting along the tension steel, the author would emphasize a further function, that of supporting the bar while it acts as a dowel. The stirrup thus transforms the dowel shear into a stirrup tensile stress instead of leaving this tension on the concrete around the bar.

In his discussion of beams subjected to multiple loads (the F-series) Dr. Taub seems to relate diagonal tension cracking to beam deflection. The author realizes that moment stresses play a large part in causing diagonal tension and that moment stresses relate to deflections. However, he has not explored the direct correlation of deflection with diagonal cracking. Time has not permitted the author to prepare load-deflection curves for publication, as suggested by Dr. Taub. Instead, the deflections at cracking loads are listed in Table B. The deflection of F2 at first diagonal cracking is larger than that of F5, as Dr. Taub assumes. Contrariwise, the deflection of F3 is larger than that of F2. It is possible that cracking load would have to be more exactly determined to make this comparison valid.

TABLE B—DEFLECTIONS

Beam	Before diagonal crack		After diagonal crack		Average deflection, in.
	Shear, lb	Deflection, in.	Shear, lb	Deflection, in.	
F2	3500	0.0602	4000	0.0732	0.0667
F3	5000	0.0682	6000	0.0876	0.0779
F5	3500	0.0482	4500	0.0694	0.0588
F7	4500	0.0520	5500	0.0718	0.0619

Classification of the failure of these beams on the basis of the sketches presented in the paper is difficult since each showed separation complete both at top and bottom. Beam F3 appeared at the time of test to be definitely a failure at the compression face with the bottom split following as a secondary result. The other three beams failed suddenly and completely in a manner that could best be described as a balanced failure; their primary failure pattern was not established.

Discussion of a paper by Cevdet Z. Erzen:

An Expression for Creep and Its Application to Prestressed Concrete*

By KIYOSHI OKADA and AUTHOR

By KIYOSHI OKADA†

The writer agrees that the creep of concrete and the change of modulus of elasticity of concrete with time are the most significant factors influencing the behavior of prestressed beams; the other most influencing factor is the shrinkage of concrete.

Mr. Erzen presents the analysis of the losses in prestressed beams in which creep and variation of modulus of elasticity with time are taken into account and the solution is obtained by numerical integration.

The writer feels that the method presented is not necessarily simple nor satisfactory enough to calculate the loss of prestress compared with the computing method conventionally used in Germany or Japan.

Eq. (4) giving the loss of tension in the wires upon transfer is derived entirely from the assumption that the unit strain in the steel at the position of the center of gravity of wires is equal to that in concrete at the same position. This assumption is not always correct, especially if the wires subjected to various magnitudes of stress before transfer lie in multiple layers of different distances from the centroid axis of concrete section.

In such a case the acting point of the resultant forces of the wires may be in a different location from the center of gravity of the wires, and the wires may be subjected immediately on transfer to bending moment besides axial tension.

Thus, as shown in Fig. A, the concrete section is subjected to axial compression, $P_{co} = P - P_{tr}$, and bending moment $M_{co} - M_D$; and the steel wires to axial tension, $P_{so} = P - P_{tr}$, and bending moment M_{so}. M_{co} and M_{so} are the bending moments acting on the concrete section and the steel wires upon transfer, respectively.

Then the following equations are obtained.

$$P_{co} = P_{so} = P - P_{tr} \quad \text{(I)}$$

$$P_{co}\, e = M_{co} + M_{so} - M_D \quad \text{(II)}$$

$$\frac{M_{co}}{E_K I_c} = \frac{M_{so}}{E_s I_s} \quad \text{(III)}$$

*ACI Journal, Aug. 1956, *Proc.* V. 53, p. 205. Disc. 53-11 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings*, V. 53.

†Assistant Professor, Kyoto University, Kyoto, Japan.

$$\frac{P}{E_s A_s} = \frac{P_{co}}{E_K A_c} + \frac{P_{so}}{E_s A_s} + c\,\frac{M_{co} - M_D}{E_K I_c} \quad \text{.....(IV)}$$

where I_s = moment of inertia of steel wires with respect to centroid axis of steel wires

Solving for P_{co}, P_{so}, M_{co} and M_{so}

$$P_{co} = P_{so} = P - P_{ir} = \frac{\dfrac{P}{D_s} - \dfrac{M_D c}{K_c + K_s}}{\dfrac{1}{D_c} + \dfrac{1}{D_s} + \dfrac{c^2}{K_c + K_s}} \quad \text{.....(V)}$$

$$M_{co} = \frac{P_{co} c + M_D}{1 + K_s/K_c},\ M_{so} = M_{co}\frac{K_s}{K_c} \quad \text{.....(VI)}$$

where $D_c = E_K A_c$, $D_s = E_s A_s$, $K_c = E_K I_c$, and $K_s = E_s I_s$

When the steel wires are in a horizontal layer c distance from the centroidal axis of concrete section, and so I_s can be assumed as zero, the above equations coincide with the author's Eq.(4). Consequently Eq.(4) is correct only in case the steel wires lie in a horizontal level, but may be used approximately in the other cases even when the effect of creep of concrete is taken into account in the second stage of the prestress loss as stated by the author.

The loss of stress due to creep of concrete can be calculated by assuming the ideal curve for creep of concrete as proposed by Whitney. The total net strain of concrete subjected to a sustained stress which varies with time, when the change of modulus of elasticity of concrete with time is taken into account, can be expressed as follows:

$$\delta_t = \frac{\sigma(t)}{E_K}(1 - q_t) + \int_K^t \frac{\sigma(\xi)}{E_K}\frac{d\,\phi(\xi)}{d\,\xi}\,d\,\xi + \int_K^t \frac{\sigma(\xi)}{E_K}\frac{d\,q(\xi)}{d\,\xi}\,d\,\xi \quad \text{.....(VII)}$$

in which the modulus of elasticity E_t is assumed to vary in such a way as

$$E_t = \frac{E_K}{1 - q_t}$$

q_t = a function of time

and $\phi(t)$ is a function of time as shown by $\phi(K/t)$ in the paper.

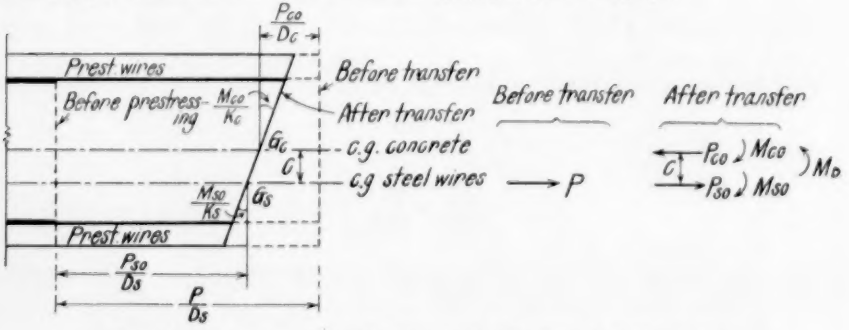


Fig. A—Strains in section before and after transfer of prestress

Applying these equations instead of Eq. (I) and (II) and assuming $q_t = k\,\phi_t$ (k a constant), an analysis similar to the author's gives the following expressions for the loss in the second stage.

$$P_t = (P - P_{tr})\left\{1 - (1 - \alpha' k\,\phi_t)^{1/k}\right\} \quad \text{(VIII)}$$

and

$$P_L = P_{tr} + P_t \quad \text{(IX)}$$

where $$\alpha' = \frac{n_K p \left(1 + \frac{c^2}{r^2}\right)}{1 + n_K p \left(1 + \frac{c^2}{r^2}\right)}$$

When there is no change in E_t, that is k is equal to zero, P_t becomes

$$P_t = (P - P_{tr})(1 - e^{-\alpha' \phi_t}) \quad \text{(X)}$$

In the above analysis no assumption is made of the form of function $\phi\,(t)$, except for constant k relating to the variation in the elastic modulus of concrete with time, and the solutions are quite simple because of no necessity for numerical calculation as made in the author's method.

When applying the above method to the same prestressed beam as given by the author, k is found to be 0.5214 and P_t is computed as 41,400 lb. Thus the total loss when $t = \infty$ is $P_L = 121{,}400$ lb.

Similarly the loss $P_L(t)$ on the succeeding days after transfer can be computed, and is plotted in Fig. B, in which the author's result is also shown for comparison.

The method described above gives a smaller value of total loss than the author's method, probably because the ideal curve for creep of concrete expresses smaller creep value, especially when the sustained loading begins at comparatively early age of concrete.

For a similar computation of the second example, the total loss when $t = \infty$ is found to be $P_L = 86{,}830$ lb in place of 91,500 lb found by the author. The discrepancy is about 6 percent but the writer's formulas are far simpler. Furthermore, the effect of variation only in modulus of elasticity of concrete upon the loss in prestress should be shown numerically in the author's paper; otherwise the effect may be sometimes over- or underestimated when the conventional computation is carried out without taking into account the variation in the elastic modulus.

According to the writer's formula, when assuming a constant modulus, the total losses in the prestress when $t = \infty$ are $P_L = 121{,}090$ lb in the first example and $P_L = 87{,}950$ lb in the second. These show that the effect of change in elastic modulus with time on the loss is comparatively small, especially when the concrete is of comparatively older age and of sufficient

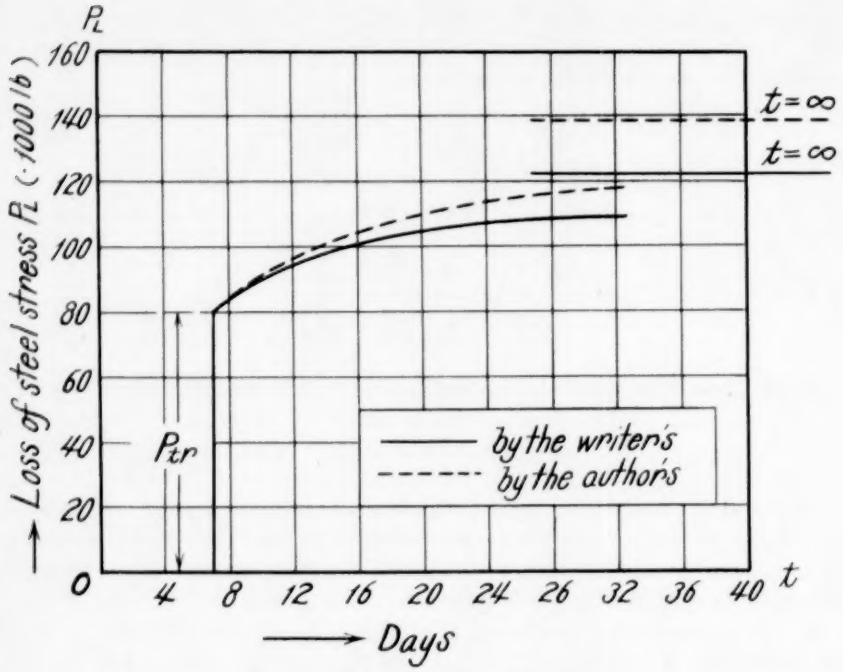


Fig. B—Variation of stress with time

strength at time of loading. Creep tests made by the writer on reinforced concrete columns having steel ratios of 1.18 and 3.72 percent, which were subjected to sustained loads at 4,14, or 27 weeks, coincides with the theoretical analysis described above. Consequently the writer believes that more tests should be made on the effect of creep, the variation in elastic modulus, and the shrinkage of concrete on the total loss in prestressed beams.

AUTHOR'S CLOSURE

The author is pleased to see Professor Okada's demonstration of the solution of creep action in prestressed concrete beams.

The equations representing creep and change of modulus of elasticity of concrete used by Professor Okada render a direct solution of the integral equation. The difference in numerical results of about 6 percent is a negligible amount.

The derivation of Eq. (5) is based on the concept of creep phenomenon as given in Reference 4, and does not contain any assumptions. Furthermore the use of Eq. (6) to (12) is quite simple. It is not necessary to resort to a numerical integration if these equations are used.

In closing, the author expresses his thanks to Professor Okada for contributing his analysis of the creep action in prestressed concrete beams.

Discussion of a paper by Myron L. Goral:

Empirical Time-Strength Relations of Concrete*

By O. LARSEN and AUTHOR

By O. LARSEN†

The hyperbolic time-strength relationship demonstrated by Mr. Goral was observed by the Illinois Division of Highways in 1934 and used in the study of test data during the intervening years. Limited publicity was given to this through several departmental reports. Mr. Goral is to be commended for giving the subject general publicity through his excellent paper.

Uniformity of the materials, of the production of the concrete, and in the making and curing of the specimens undoubtedly is the prerequisite for successful estimates of future strength. Bearing this in mind, the use of the theory of least squares, which gives equal weight to all points on the rectified strength curve, may not be advantageous in preliminary estimates of future strength. It would be of greater value to consider the circumstances with respect to the specimens representing the strength tests. For example, unless the specimens can be placed in standard curing immediately after they are made, the time they remain in the forms constitutes a considerable percentage of the curing period insofar as early strength is concerned, and may affect adversely or favorably the strength obtained. If in the rectified graph the points plotted do not fit a straight line, the circumstance described may be sufficient reason for disregarding an early strength value and for determining the position of the line entirely on the basis of other points, even if only two are available. In a rather extensive series of tests of various Type III cements, conducted by the Illinois Division of Highways, it was observed that the 1-day strengths invariably did not quite conform to the linear relationships while those at later ages generally did.

A further prerequisite for successful estimates of future strength is accuracy in determining the strength values, as well as the test ages, that form the basis for the estimates. Tests of only one or two specimens at each test age, as too often is the case, would not be expected to yield satisfactory data. It is essential that the number of specimens be sufficient to provide average strength data from which undue errors of testing are eliminated.

The Illinois Division of Highways has further developed the method, and some of the additional equations derived may be of interest. Since the relationship applies also to flexural strength, Eq.(1) may be written as follows:

$$f' = \frac{t}{b' + m't} \qquad \text{(A)}$$

*ACI JOURNAL, Aug. 1956, *Proc.* V. 53, p. 215. Disc. 53-12 is a part of a copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Engineer of Mixtures Control, Illinois Division of Highways, Springfield, Ill.

where the prime has been inserted to denote values pertaining to flexural strength.

Solving Eq. (1) and (A) simultaneously for f and f', the following values are obtained:

$$f = \frac{f'}{p + qf'} \quad \text{(B)}$$

and

$$f' = \frac{f}{\frac{1}{p} - \frac{q}{p} f} \quad \text{(C)}$$

In terms of intercepts and slopes of the rectified curves representing corresponding compressive and flexural strengths,

$$p = \frac{b}{b'} \qquad \text{and} \qquad q = m - \frac{b}{b'} m' = m - pm'$$

However, the values of the constants may more easily be determined analytically by Eq. (B) from two pairs of corresponding compressive and flexural strength values, accurately determined. It is of interest that the equations giving the relationship between compressive and flexural strengths are of the same form as Eq. (1).

Further, with reference to Eq. (1), if f_1 represents the compressive strength at the age t_1, then the existing relationship is given by the equation

$$f_1 = \frac{t_1}{b + mt_1} \quad \text{(D)}$$

Dividing this expression by the expression for f in Eq. (1) to obtain the ratio of f_1 to f and solving for f_1, the following equation results:

$$f_1 = \frac{ft_1 (c + t)}{t (c + t_1)} \quad \text{(E)}$$

In terms of intercept and slope of the rectified curve representing the compressive strength,

$$c = \frac{b}{m}$$

The value of the constant may also be determined analytically from Eq. (E) from accurately determined strength values at two ages. If strength data at early ages are known and the constant can be evaluated satisfactorily, this equation is convenient for estimating the strength at a later age, and applies in the case of either compressive or flexural strength data.

In similar manner, equations were derived expressing the compressive strength in terms of the flexural strength at the same and at different test ages. These involve three constants that also can be expressed in terms of intercepts and slopes of the rectified strength curves. Inasmuch as they have not been found particularly useful, they will not be stated here.

AUTHOR'S CLOSURE

The comments of Mr. Larsen are well taken. The additional relation of flexural strengths and the relation of flexural to compressive strength is indeed an important tool in estimating later strengths from early test cylinders.

One point, however, needs further clarification. The validity of very early strengths (ages 1 to 3 days) can be maintained only if the age is measured with the same accuracy as the strength. Experience has shown that if the age of an early test cylinder is accurately recorded in hours, and if this age is divided by 24 to get the age in days to two decimal places, 90 percent of the 1- and 2-day tests will lie on the rectified line as do the later tests. Since the slope of the time-strength curve is steep at early ages, the rectification procedure is more sensitive to errors of time measurement than to strength measurements. After about 10 days, the day is a sufficiently accurate unit of measurement for most practical applications.

Discussion of a paper by A. C. Scordelis, K. S. Pister, and T. Y. Lin:

Strength of a Concrete Slab Prestressed in Two Directions*

By JACK JANNEY, GROVER L. ROGERS, and AUTHORS

By JACK JANNEY†

Messrs. Scordelis, Pister, and Lin are to be commended on a clear-cut investigation and presentation of test results. It is refreshing to read a research paper in which the objectives are clearly outlined and adhered to, and only pertinent data are presented. Test information regarding the behavior of prestressed slabs is scarce and is needed.

The method of loading the test member devised by the authors was well conceived. Too few tests are conducted in which a uniform loading is applied. However, the indeterminate nature of the slab supported on the four corners presents distinct possibility of error in analyzing test results. Anything short of the care employed in this test could result in questionable data. In fact, the authors were fortunate to obtain the uniform results reported. The corner reactions could have been made determinate inexpensively by utilizing a small hydraulic ram and calibrated compression load cell at each corner. In this way the four reactions could have been maintained equal in a positive manner throughout the test series. In addition, it would have been possible to vary the distribution of reaction for several tests in which the loadings were such that the slab behaved elastically. The reactions at the four corners of a lift slab will seldom be equal in the field. If the above system of reaction measurement were used, accurate control of these tests would be possible.

It is unfortunate that the prestressing reinforcement in relatively small slabs cannot be bonded economically. The behavior after cracking of a bonded member is superior to that of an unbonded member.

It is rather dangerous to assume that the steel stress at bending failure in an unbonded member is equal to "80 to 90 percent of the ultimate steel strength." The steel stress in a unit in which the prestressing reinforcement is unbonded is much more dependent upon the effective prestress than it is in a bonded

*ACI Journal, Sept. 1956, *Proc.* V. 53, p. 241. Disc. 53-13 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.
†Consulting Engineer, Chicago, Ill.

member.* Of course, the final stress in an unbonded member is also dependent to some extent on the percentage of steel, span of the member, and manner of loading. It is possible for the steel stress at failure to be much less than the 80 to 90 percent of the ultimate tensile strength suggested. The value of 210,000 psi determined for the steel stress at failure in the test slab is 84 percent of the tensile strength of the steel. Yet, the authors recognized that this value was higher than it would have been if the prestressing steel were completely unbonded. In fact, it should have been about 175,000 psi which is 72 percent of the steel strength. The high steel stress actually determined has been attributed to friction between the concrete and steel which caused a stress differential along the length of the prestressing tendons when the slab was deflected appreciably. Steel stresses considerably below the 72 percent may frequently occur at failure.

By GROVER L. ROGERS†

The authors have contributed valuable experimental information on the subject of cracking loads to be expected when dealing with concrete slabs which have been prestressed in two directions. It is gratifying to note that their conclusions are essentially the same as those drawn by the writer‡ from tests performed in 1951-52.

The first experimental results obtained on slabs prestressed in two directions were presented by Freyssinet§ who concluded that the load causing cracking could not be predicted with any degree of accuracy by using the classical theory of elasticity. The test slab discussed by Freyssinet was a runway pavement slab at the airfield at Orly, France. It has been pointed out‡ that the cracking load measured by Freyssinet was not the "first" cracking load but was rather the ultimate cracking load. This is evident in that one could not observe first cracking for it would have occurred on the underside of the slab. Levi** has conducted similar tests in Italy on such pavements and found that this ultimate cracking load could be predicted if the plasticity of the concrete were considered.

The writer‡ conducted a test on a square slab subject to two concentrated loads and found that the loads causing first cracking were not greatly different from those predicted using elasticity theory. Tests performed by Magnel** and Lin** on indeterminate beams gave essentially the same results.

Guyon** reported that tests performed by him on continuous slabs indicated that the first cracking load was greatly different from that predicted

*Gifford, F. W., "The Design of Simply Supported Prestressed Concrete Beams for Ultimate Loads," *Proceedings*, Institution of Civil Engineers (London), V. 3, Part III, Apr. 1954, pp. 125-143.
Janney, J., Hognestad, E., and McHenry, D., "Ultimate Flexural Strength of Prestressed and Conventionally Reinforced Concrete Beams," ACI JOURNAL, Feb. 1956, *Proc.* V. 52, pp. 601-620.

†Professor of Civil Engineering, Virginia Polytechnic Institute, Blacksburg, Va.

‡Rogers, G. L., "Validity of Certain Assumptions in the Mechanics of Prestressed Concrete," ACI JOURNAL, Dec. 1953, *Proc.* V. 50, pp. 317-330.

§Freyssinet, E., "Importance et Difficultes de la Mecanique des Betons," *Proceedings*, First International Congress on Prestressed Concrete, Ghent, Belgium, 1951.

**Levi, F., Guyon, Y., and Magnel, G.: discussions of "Validity of Certain Assumptions in the Mechanics of Prestressed Concrete," by Grover L. Rogers, ACI JOURNAL, Dec. 1954, Part 2, *Proc.* V. 50, pp. 332.1-332.4

using elasticity theory. In discussing the writer's paper, he emphasized the importance of "secondary carrying systems" and suggested that the difference between calculated and measured cracking loads would increase as the number of such secondary systems was increased. A square slab uniformly loaded (as was the case studied by the authors) has four axes of symmetry. If a rectangular slab had been treated, there would have been only two axes of symmetry and consequently the number of "secondary carrying systems" would have been increased, and according to the Guyon hypothesis the difference between predicted and measured cracking load would have been correspondingly increased.

It should be pointed out that the slabs studied by the authors and by the writer were given supports which could be quite accurately treated using classical theory. The slabs tested by Guyon could be treated only approximately using the classical theory.

The writer hopes that this latest work performed by the authors will inspire others to undertake additional experimental investigations in what is an important area of reinforced concrete design.

AUTHORS' CLOSURE

The authors wish to thank Messrs. Rogers and Janney for contributing constructive and informative discussions to the paper.

The authors agree with Professor Rogers that in all probability the cracking load observed by Freyssinet on the runway pavement slabs at Orly, France, was the ultimate crushing load rather than the first cracking load. It is difficult to detect by visual inspection the initial cracking of a prestressed concrete structure. The load at first cracking can often be better determined by plotting experimental load-strain curves for critical points. In almost all cases these plots will indicate a linear relationship up to first cracking after which the strains will increase or decrease more rapidly with load depending on whether the crack occurs within or just outside the strain gage. Together with the results reported earlier by Professor Rogers,* the test results obtained by the authors substantiate the premise that the theory of elasticity may be used to predict stresses up to initial cracking.

Mr. Janney's suggestions regarding ways in which the testing technique could have been improved are excellent. The authors were aware of the sensitivity of the structure because of its external indeterminacy and great care was taken to insure equal reactions at the four supports by the method described in the paper. The use of a hydraulic jack or ram resting on a load cell at each support would have been a means by which the value of each reaction could have been determined more accurately. Perhaps if equal reactions are desired it would be better to have all four jacks or rams supplied from the same hydraulic line rather than operating each independently as indicated by Mr. Janney.

*Rogers, G. L., "Validity of Certain Assumptions in the Mechanics of Prestressed Concrete," ACI JOURNAL, Dec. 1953, *Proc.* V. 50, pp. 317-330.

Almost all of the lift slabs built on the Pacific Coast have unbonded cables. Many of these slabs have prestressing units similar to those used in the test slab. It is difficult to predict the steel stress existing in an unbonded cable under the ultimate load condition. Naturally if the effective prestress in the cable were very low it would not be possible to develop "80 to 90 percent of the ultimate steel strength." The amount of wire stretch necessary could probably not occur before failure. This is also true to a much lesser degree in a bonded cable. Lacking any better information the authors used data obtained from earlier tests in the laboratory on unbonded beams having the same effective prestress, to estimate the stress in the cables at ultimate load. It would be advisable in future tests to experimentally determine strains in the prestressing cables at controlling points so that a better estimate of the stress at failure might be obtained.

Discussion of a paper by Eric C. Molke:

Auditorium Framed With Prestressed Roof Girders*

By DAVID P. BILLINGTON, FRED E. KOEBEL, and AUTHOR

By DAVID P. BILLINGTON†

This paper presents a most interesting project in which the construction technique of raising a large prestressed girder between two sets of twin columns has been successfully applied on the American scene. There are several design considerations which Mr. Molke has brought up and which merit further discussion.

First, is the question of the design of such a large-span structure as a rigid frame. Mr. Molke points out that it is up to the designer not only how he shall make the rigid frame where the main girder is precast on the ground, but also at what stage in the construction this rigidity shall be built in. The method described in this paper for achieving continuity between the precast girder and the cast-in-place columns consists simply of prestressing the girder and the column top together. This is done by inserting cables into preformed holes through the column top which are lined up with preformed holes in a thickened portion at each end of the girder. These prestressing units are only 8 ft long. With regard to the method of designing the frame, the writer would like to suggest that an alternate method is also quite applicable to such structures as the one described by Mr. Molke. This also involves a two-stage prestressing operation as was done in this case but it incorporates in the second stage of prestressing several advantages which the method described in the article does not have.

The method is simply to leave some of the prestressing tendons which run the full length of the girder unprestressed until the girder is erected. As soon as the girder is in place these unprestressed tendons would then be prestressed against the column and thereby create the rigid frame. In this manner, the prestressing steel which is already in the design performs the second function of tying the girder and columns together. Furthermore, the second stage of prestressing can also have the effect of prestressing the columns and the foundations. The second-stage prestressing force can be applied by means of tendons draped in a parabolic manner throughout the entire girder length. These tendons would be prestressed after the girder is in its final location on the columns. The column top would have to be de-

*ACI Journal, Oct. 1956, *Proc.* V. 53, p. 363. Disc. 53-19 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.
†Roberts and Schaefer Co., New York, N. Y.

tailed so that these tendons could be blocked against the columns themselves. However, the difference would be that the second-stage prestressing force which is used to connect the girders and the columns would also cause a rotation at the junction of these members. Because the shape of the prestressing tendons is parabolic, the bending moment line will also be parabolic. This bending moment line is therefore of the same shape as that due to the superimposed loads, but in the opposite direction. The rotation caused by this prestressing force is thus opposite to the rotation caused by dead and live loads. This means that any force induced in the columns through frame action due to dead and live loads will be counteracted by the second-stage prestressing forces. In this way, the undesirable bending moments and horizontal thrusts described by Mr. Molke could be neutralized by this frame prestressing.[1] Also, this would mean that it would be possible to design the structure as a frame not only for live loads but also for the dead loads of the roof slabs. This would, of course, require a slightly different detail at the top of the columns to allow this second stage of prestressing.

The second design factor mentioned by Mr. Molke is the question of stability both for the prestressed girder alone and for the completed frame structure. Mr. Molke correctly points out that a prestressed girder can easily be subject to buckling just as other reinforced concrete members from external loads. The impression that a prestressed member will not buckle is a dangerous one when incorrectly applied. However, it is still true, as Magnel points out in his book,[2] that a prestressed member will not buckle under the prestressing load if the prestressing steel is in contact with the concrete at a sufficient number of points along the length of the member. This applies as well to lateral buckling as it does to vertical buckling. Therefore, in post-tensioned members, it is necessary to have the tendons in contact with the girder such as is surely the case where the stressed units are draped.

It is nevertheless important that there be enough lateral stiffness in a precast member to resist buckling due to dead and live loads. In precast design, handling and erection often produce the most critical conditions for stability. In the case of these auditorium girders this problem is minimized by the controlled nature of slow lifting between rigid supports. It may be of interest to point out that 136-ft girders of nearly this size have recently been erected for a hangar at Hill Air Force Base, Ogden, Utah, where the lifting was done solely by cranes.[3] The ratio of flange width to free span was 38 in these girders, as compared with 36 in the girders described by Mr. Molke. Just as in the case of the auditorium girders, the width was kept to a minimum to avoid adding materials and lifting weight.

The question is raised concerning the danger of accidentally supporting the girder at the center of its span during handling. This would produce somewhat the same effect as if the girder were overturned since the girder dead load would now add to the prestressing load and cause high overstressing. The danger of failure here is due to high bending stresses and not necessarily connected to a buckling problem.

With regard to the lateral stability of the completed building, Mr. Molke calls attention to the similarity between this structure where the precast roof members are connected to the girder bottom flange and a barrel shell roof with stiffening ribs protruding above. A shell roof is monolithic with the bottom of the arch and provides lateral support even for the top of the arch. This will be the case with the precast girder too. The only difference is that the girder bottom flange and precast roof panels are not fully monolithic in the structure described in the paper. The connection is more like a hinge. The question which needs further investigation is, therefore, how effective will such support be as compared with the monolithic arch and shell?

REFERENCES

1. Billington, D. P., "Building Frames in Prestressed Concrete," ACI JOURNAL, June 1956, *Proc.* V. 27, pp. 1065-1081.

2. Magnel, G., *Prestressed Concrete*, Concrete Publications Ltd., London, 3rd Edition, 1954, Chapter VII.

3. "Air Force Hangar Uses 136-ft Precast Beams," ACI JOURNAL, Sept. 1956, *News Letter*, pp. 7-10.

By FRED E. KOEBEL*

Mr. Molke's paper was read with interest. The writer's firm, Prestressing, Inc., was subcontractor for the construction and erection of the girders, to the firm of J. E. Pyle General Contractor, general contractor on the job.

This type of construction definitely shows the use of prestressing for long-span girder construction. The use of twin columns eliminated the necessity of heavy lifting equipment and provided the means for the continuity stressing after the girder was in place. The use of this continuity stressing shows definite economies in this type of long-span girder construction and it was, it is believed, the first use of this application in the United States.

Since this construction has been completed, two other jobs of similar design and construction technique have been completed. One job has girders of 100-ft span and the other job has girders with total lengths of up to 110 ft. In this job the clear span was approximately 95 ft with a cantilever of 15 ft. The joints were again made continuous by post-tensioning after erection. In this job, incidentally, because of the location of the girders it would have been practically impossible to erect them with cranes. Thus the system of erection outlined in Mr. Molke's paper was utilized to place these girders without excessive erection costs.

In this paper Mr. Molke has recorded a fascinating application of precasting and prestressing. The job shows the results of excellent coordination between architect, structural engineer, and contractor.

*Vice-President and General Manager, Prestressing, Inc., San Antonio, Tex.

AUTHOR'S CLOSURE

One can easily agree with Mr. Billington that by some operation of two-stage prestressing of the draped tendons in the girder, so-called "secondary moments" could be induced in columns and footings, thereby causing greater restrainment of the girders.

Such procedure will be more feasible for cast-in-place prestressed frame construction for which it was first conceived, but would seemingly call for cumbersome details where girders are cast on the ground and hoisted into place.

Furthermore, as was pointed out, small columns and a footing design with low horizontal thrust seemed to be desirable for reasons of economy. Under such conditions the restrainment of girder to columns can by necessity not be relied upon to reduce substantially the positive bending moments at midspan, regardless of whether the columns are prestressed or not.

Mr. Billington's statement that primary buckling failure cannot occur so long as the prestressing steel is in contact with the concrete at a sufficient number of points along the length of a member has never been doubted by the author. Naturally this goes for an action in either the horizontal or the vertical plane.

The paper, however, suggested a lateral buckling effect in the thin webbed member coupled with the stress action in the vertical plane "similar to the torsional buckling in narrow I-beams." For the same reason it was pointed out in the paper that an angular motion of a supporting base during the construction condition could trigger such torsional or secondary buckling.

In regard to Mr. Billington's query as to the degree of stiffness of the connection between roof slab and girder, it should be noted that each precast roof panel had anchor plates welded to matching anchor plates of the roof girder and, as indicated in Fig. 9 and 10, a substantial sloping concrete corner fill rammed into the space between panel ends and girder face was continuously applied at both sides of a girder. The bending strength of this connection was calculated to be greater than the bending strength of the roof panels and such connection can certainly not be classified as a hinge.

The author is particularly grateful to Mr. Billington for the statement that he has already used a greater ratio of free span to flange width. As Mr. Koebel has pointed out, prestressed concrete frame construction will become very useful for longer spans, and the question of permissible span-width ratio is in need of a realistic approach.

Discussion of a paper by Ralph W. Kluge:

Structural Lightweight-Aggregate Concrete*

By WALTER K. WAGNER and AUTHOR

By WALTER K. WAGNER†

Professor Kluge is to be complimented on his compilation and presentation of data of general nature that has long been needed. Basically, the structural lightweight aggregates are similar in character and performance. The great difference appears to be in the aggregate crushing strength and low absorption values for certain high strength-producing aggregates. These desirable characteristics are obviously due to sealed aggregate pores developed in the expansion process.

Pumice, as a natural product, has open, interconnected pores which allow water to penetrate the aggregate structure and produce high volume changes. Apparently, this moisture transition is never a static condition, even within the matrix of hardened concrete. The volume changes have been so great in structural pumice concrete in the Southwest that most designers have abandoned its use for large structural panels or members that might be affected by shrinkage or cracks. The shrinkage problem needs further research before pumice and similar aggregates can be used in structural design with confidence.

The control of quality of lightweight concrete mixes still is a complex procedure because of the wide range of absorption and density of the aggregate particles. One simple method of control used successfully by this writer is to consider a nominal mix on the basis of absolute volumes. These are computed from a "quick specific gravity" determined by displacing a fairly large sample of the aggregate in the condition it is to be used for the mix. Changes in moisture and particle density are reflected in this value, and the resulting yield of the mix is more stable. The adjusted weights for the mix are simply the absolute volume of the basic mix multiplied by the specific density (quick specific gravity $\times$ 62.4). Water must be adjusted by "eye" or some physical measure of consistency. This writer has never found any relationship between the water demand per cubic yard of concrete and the absorbed moisture of the aggregates. Experience indicates that water can be added for uniform workability to mixes with aggregates of highly different moisture contents without seriously affecting the strength of the concrete.

*ACI Journal, Oct. 1956, *Proc.* V. 53, p. 383. Disc. 53-21 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Chief Engineer, Albuquerque Gravel Products Co., Albuquerque, N. M.

One of our recent experiments show this on a 1:6 mix made with three batches of the same aggregate adjusted to different moisture contents before mixing. Batching was on a volume basis: ½ bag of cement to 3 cu ft (loose volume) of fine pumice. Table A gives the results.

TABLE A—PUMICE CONCRETE MIX DATA

	Batch 1	Batch 2	Batch 3
Moisture in aggregate (by dry weight), percent	0.76	11.11	48.15
Batch weights, lb			
Water	65.0	59.0	29.0
Cement	47.0	47.0	47.0
Pumice (3 cu ft)	118.5	117.9	131.5
Total	230.5	223.9	207.5
Gross *W/C* ratio	1.38	1.26	0.62
Slump, in	2	3¼	3½
Concrete unit weight, lb per cu ft	81.2	84.0	88.8
Compressive strength, psi			
7 days	567	550	600
28 days	1083	1167	1325

AUTHOR'S CLOSURE

The author and Subcommittee I of ACI Committee 213 thank Mr. Wagner for the comments in his discussion of the report. They further confirm the practice of adding sufficient water to a lightweight-aggregate concrete mix to produce a given consistency rather than to depend upon a particular water-cement ratio as a guide.

The author would also like to comment further on a statement contained in the report to the effect that the modular ratio need not play an important role in the design of flexural members of lightweight aggregate concrete.

An elaboration of this may have some merit particularly for those who are accustomed to design beam sections in the conventional manner, namely "balanced design" in which the concrete and steel are theoretically stressed to their working values simultaneously. For dense-aggregate concretes this procedure is satisfactory and accepted. For lightweight-aggregate concretes the method leads to shallow sections with higher percentages of steel because of the higher modular ratio for this type concrete. From the standpoint of deflection this is definitely undesirable. It can also be uneconomical since the cost of the reduced volume of concrete is usually more than offset by the cost of the additional reinforcement required. The obvious step is to increase the depth, which decreases the stress in the concrete, the area of steel being adjusted so that it operates at its full working stress. The question then arises, what is the value of j to use for adjusting the steel area? If an exact value of j is wanted perhaps the simplest method is a trial-and-error process employing the transformed section. Actually it will generally be found to vary but a few percent from the theoretical value based on balanced design. This is illustrated by the curves of Fig. A which were obtained for sections 10 and 14 in. in width and for depths increased from the balanced design depth by the amounts shown as abscissa. The bending moments used for this illustration were 600 to 1200 in.-kip. Values of j are shown for two

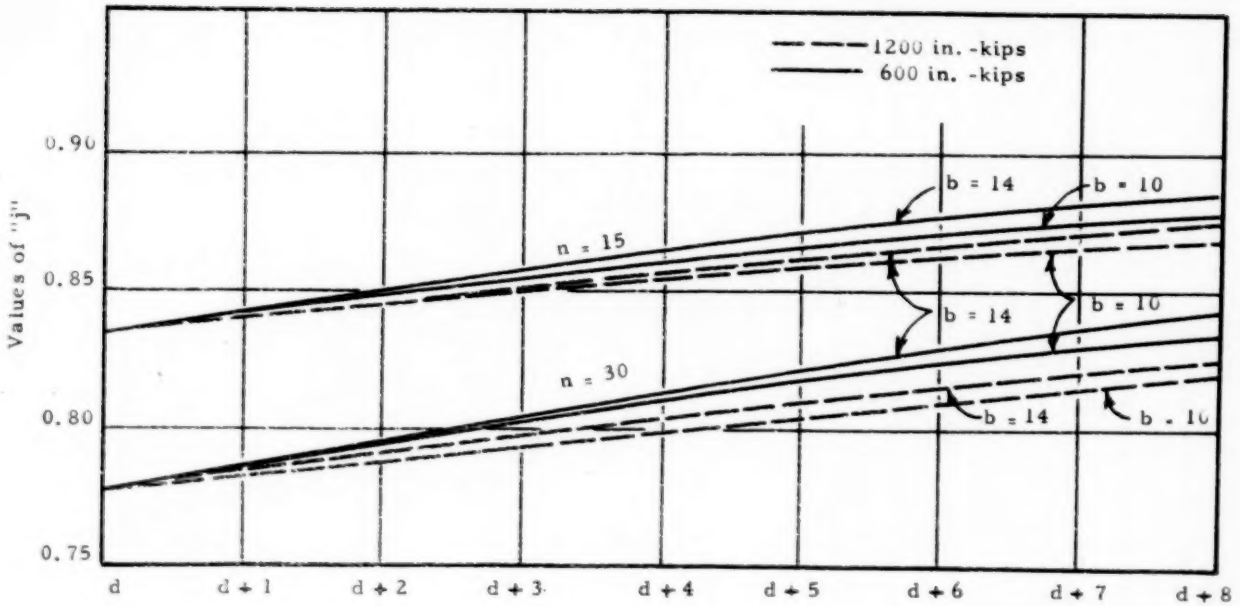


Fig. A—Variation of *j* with depth greater than theoretical

extreme values of the modular ratio, $n = 15$ and $n = 30$. The curves show that for a 5-in. increase in effective depth the maximum change in j is less than 5 percent for a given modular ratio. It is evident then that the value of j for balanced design could be used for reasonable increases in depth without serious error.

Another interesting fact brought out by these data is the relatively little effect that a rather wide difference in the modular ratio has upon the value of j. The maximum difference indicated by these curves is less than 7 percent for any given set of conditions. It would appear that an exact value of the modular ratio is unimportant particularly if the effective depth is greater than the so-called theoretical depth. When it is considered that the elastic modulus of concrete in a structure is not known with any degree of certainty and that the value of E changes with age as well as with plastic flow, there is no justification for requiring an exact value of n.

Discussion of a paper by Victor R. Bergman:

Helicoidal Staircases of Reinforced Concrete*

By H. S. GEDIZLI, HELGE HARBOE, FRITZ LEONHARDT, WILLIAM T. MARSHALL, JOSÉ SANABRIA, A. C. SCORDELIS, and AUTHOR

By H. S. GEDIZLI†

Many methods of analysis of helicoidal staircases have been published[1] and Mr. Bergman's paper is one of them. However, it may be deemed advisable to forego the simplifications offered by these methods, and to follow Dr. Fuchssteiner's theory[2] and the practices born from it.[3]

REFERENCES

1. Malcor, R., "Théorie de l'Équilibre de la Résistance des Escaliers à Vis," *Travaux* (Paris), Dec. 1948, p. 605.
   Measor, E. O., and New, D. H., "Design of a Helical Staircase," *Concrete and Constructional Engineering* (London), V. 46, No. 9, Sept. 1951, pp. 283-286.
   Halbritter, F., "Berechnung gewundener Treppenlaeufe," *Beton- und Stahlbetonbau* (Berlin), V. 47, No. 7, July 1952, p. 159.
   Schröder, H., "Eine freitragende Wendeltreppe aus Stahlbeton in Belgien," *Beton- und Stahlbetonbau* (Berlin), V. 47, No. 9, Sept. 1952, p. 219.
   Cohen, J. S., "Design of Helical Staircases," *Concrete and Constructional Engineering* (London), V. 50, No. 5, May 1955, pp. 181-194.
   Mattock, A. H. "Design and Construction of a Helical Staircase," *Concrete and Constructional Engineering* (London), V. 52, No. 3, Mar. 1957, pp. 99-105.
2. Fuchssteiner, W., "Die freitragende Wendeltreppe," *Beton-Kalender* (Berlin), Part 2, 1955, p. 189.
3. Fuchssteiner, W., "Die freitragende Wendeltreppe (Spannungsberechnung und Bewehrung)," *Beton- und Stahlbetonbau* (Berlin), V. 49, No. 11, Nov. 1954, pp. 256-258.
   Discussion: "Die freitragende Wendeltreppe," by H. S. Gedizli and W. Fuchssteiner, *Beton- und Stahlbetonbau* (Berlin), V. 50, No. 9, Sept. 1955, p. 247.

By HELGE HARBOE‡

The author states on p. 406 that it is unnecessary to assume that the staircase is equivalent to its horizontal projection. This assumption is believed by the author to be convenient and accurate enough for the practical analysis of moderately sized helicoidal staircases.

The writer disagrees with the author, as a more realistic analysis, to which the author also refers (References 6 and 7), tends to create a staircase of more slender thickness and beauty, at the same time, much more economical.

*ACI JOURNAL, Oct. 1956, *Proc.* V. 53, p. 403. Disc. 53-22 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 8, No. 12, June 1957, *Proceedings* V. 53.
†Ankara, Turkey.
‡Civil Engineer, Rio de Janeiro, Brazil.

I will, in this respect, refer to an article treating the above subject in *Beton-Kalender*, Part II, 1955, p. 193 (Section 6.2—"The Stair Fixed in Both Ends"). It is well to translate the beginning of this article, which, in a concise form, calls attention to the principal way in which the end supports, being at different heights, acts favorably:

This type of stair is a general rule in reinforced concrete construction. It should straight away be mentioned that the fixed stair has a considerable shell effect. In other words, the great bearing capacity of the shell is based on the activity of deformation forces. Considering the slab of a stair as a shell, then M_y produces deformation forces which are distributed uniformly over the thickness of the shell. M_x, however, is a bending moment in shell theory; it means bending about the weak axis of the shell section. For the staircase, the activity of M_y produces the main part of the bearing capacity.

Compared herewith the contribution of M_x with regard to bearing capacity is insignificatn.

The spiral staircase thus differs considerably from the circular girder. By projection of the spiral staircase on the base plan and treating same as a plan circular girder, the most valuable bearing reserve would be lost. This is clearly understood by the following consideration (Fig. 26) [Fig. A].

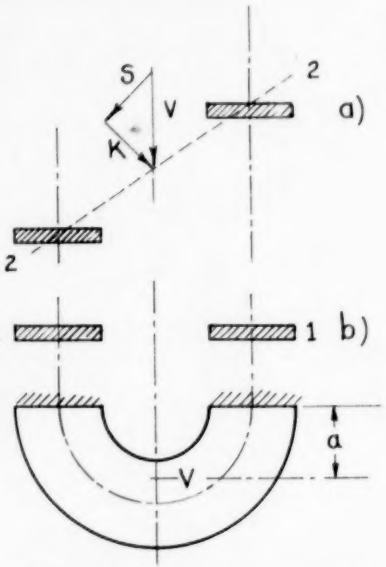


Fig. A—Fig. 26 from *Beton-Kalender*

A semicircular staircase as per Fig. 26 [Fig. A], supports the total load V which acts in the distance a from the fixed support. The support section has therefore to be calculated for the moment Va. This moment acts by the plan circular girder (Fig. 26b [Fig. Ab]) around the weak axis 1-1. By the spatial curved stairway (Fig. 26a [Fig. Aa]) the component S from V produces an insignificant frame action. The other component K gives the fixed moment Ka. *Here, however, the moment of inertia with regard to the main axis 2-2, is much more able to act.* All section elements above the inclined axis 2-2 get tension while the elements below get compression.

Therefore, it is principally the moment M_y which is acting.

By FRITZ LEONHARDT*

The paper by Mr. Bergman gives interesting data for the calculation of helicoidal staircases of reinforced concrete. I have designed several staircases with a total central angle up to 410 deg and found that not only is the calculation important but also the architectural form. Such staircases are only good looking if the outer slab depth is as narrow as possible. This can easily be reached by using a conical cross section with greater depth along the inner circle.

*Consulting Engineer, Stuttgart, Germany.

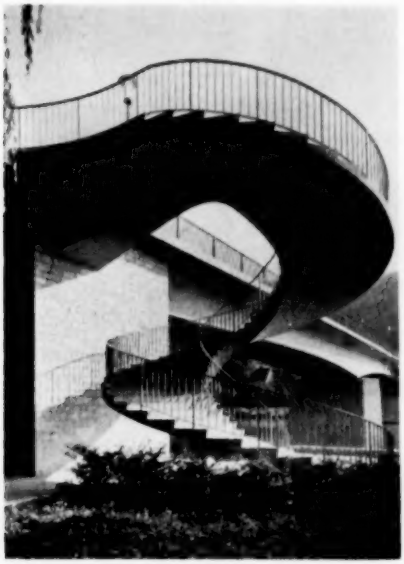


Fig. B—Neckar Bridge staircase

As an example, there is the staircase at the Neckar Bridge, Ziegelhausen, near Heidelberg, a prestressed concrete bridge which was built following my design 2 years ago (Fig. B).

By WILLIAM T. MARSHALL*

The writer is interested in Mr. Bergman's paper as he himself had worked on a similar problem some years ago.[1] Mr. Bergman has however simplified the problem by working from the center of the beam instead of the supports, thus reducing it to one of a single redundancy in the case where the load is symmetrical.

Mr. Bergman mentions the difficulty of designing the shear reinforcement but does not, in the writer's opinion, give enough information. τ_{max} and v must be added algebraically, but what value is to be taken for the shear when designing the reinforcement? To take $(\tau_{max} + v)$ acting over the whole section is obviously wrong. From Fig. 4 it would appear that the torsional reinforcement required should be sufficient to resist a force of $\frac{1}{2}\ \tau_{max} \times b \times h/2$ in a direction parallel to each long side. The value of v at sections corresponding to τ_{max} is zero. If the shear parallel to each short side is considered, then each arm of the stirrup provided should be designed to carry $V/2 + (\tau_{zy} \times$ area over which this force acts). From Fig. 4 the value for this area would appear to be $2h/3 \times b/4 \times 1/3$ (assuming a parabolic stress variation acting over a quarter of the width).

*Regius Professor of Civil Engineering, James Watt Engineering Laboratories, University of Glasgow, Glasgow, Scotland.

It will be found however that each method gives binders at very close spacing.

The writer carried out a number of experiments on reinforced concrete beams in torsion[2] and these showed that up to at least 50 percent of the ultimate load the beam behaved as a plain concrete beam. The theoretical analysis used by Mr. Bergman for calculating the torsional stresses is based on a monolithic uncracked section. Use of steel reinforcement stressed to its normal working value implies cracks in the concrete. Under these conditions the theoretical analysis given will not hold and the writer feels that the section should be made so that it can resist the calculated torsional moment without any reinforcement.

The ultimate torsional resistance of an unreinforced section can be obtained using the "sand heap" analogy and taking the ultimate tensile stress as the maximum torsional stress.[3] If this is represented by t then the ultimate torsional moment, T, of a section of width b and depth h is given by

$$T = t\left[\frac{h^2}{2}\left(\frac{b-h}{2}+\frac{h}{3}\right)+\frac{h^3}{6}+\frac{h^2}{4}(b-h)\right]$$

An appropriate load factor must be applied to this to give the safe torsional moment which can be resisted by the section. If t is taken as 200 psi—a reasonable value for the average concrete—then the ultimate torsional moment of the 72 x 10-in. section used in Mr. Bergman's example is 57,200 ft-lb. The applied torsional moment is 12,600 ft-lb, giving a load factor of 4.54, a value which in the writer's opinion shows that no torsional reinforcement is required.

REFERENCES

1. Marshall, W. T., "Design of Bow Girders in Reinforced Concrete," *Concrete and Constructional Engineering* (London), Dec. 1941 and Apr. 1942.

2. Marshall, W. T., "Experiments on Plain and Reinforced Concrete in Torsion," *The Structural Engineer* (London), Nov. 1941.

3. Marshall, W. T., "The Ultimate Torsional Resistance of Plastic Materials with Special Reference to Concrete," *Concrete and Constructional Engineering* (London), Apr. 1944.

By JOSÉ SANABRIA*

The method developed by Mr. Bergman of projecting the helicoidal staircase in a horizontal plane and calculating the bending and torsional moments and shearing stresses as in a curved beam loaded normal to its plane is not new and has been treated by various authors in the past. As an example, the book, *Cálculos de Construcción*, by M. Company (G. Gili S. A. publisher, 1948), contains in pp. 502-522 a complete description of this method, with tables of bending and torsional moments illustrating a particular case of a helicoidal staircase.

*Engineer, Caracas, Venezuela.

Fig. C—Helicoidal concrete staircase in Caracas

We know of many helicoidal staircases that have been designed using this method. Fig. C shows a staircase designed by this method by Luis Báez Díaz and constructed in his house in Caracas.

The method is only an approximation, but generally sufficient for the majority of cases in actual practice.

The graph in Fig. 2, as well as the recommendations for determining the reinforcement, will be useful in practice.

Other authors have from time to time attempted a more exact analysis of this type of staircase. Among the published papers that we know of, one which deserves special mention is by L. Sobrero, "Calcolo di Verifica di Una Struttura Elicoidale Iperstatica," *L'Ingegnere* (Rome), V. 8, No. 8, Apr. 15, 1934.

By A. C. SCORDELIS*

The author has presented an interesting and simple approach to the design of a helicoidal staircase. The writer has been interested in the analysis and design of helicoidal staircases for a number of years since he has on occasion had the opportunity to design such structures. Extensive studies have been made and are being conducted at present at the University of California on the analysis of helicoidal girders.

In the method presented by the author the simplifying assumption was made that sufficiently accurate results could be obtained by analyzing the horizontal projection of the helicoidal girder. This reduced the problem to the analysis of a bow girder under uniform load, a relatively simple problem. While this simplification may be legitimate for design purposes it is still important to know, to some extent, what degree of approximation is involved in the assumption. All subsequent calculations to determine stresses and reinforcement requirements are dependent on the internal forces and moments found in this analysis.

*Assistant Professor of Civil Engineering, University of California, Berkeley, Calif.

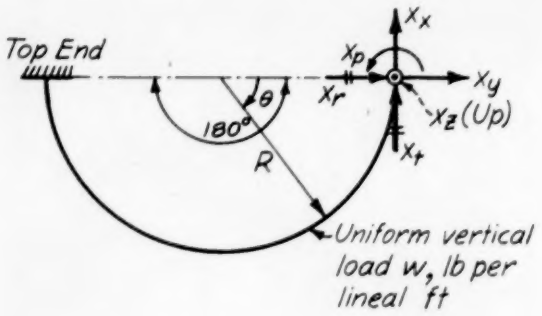


Fig. D—Redundants at bottom end of helicoidal girder (Use right-hand rule for moment vectors)

It is the purpose of this discussion to compare the results obtained using the above assumption and those obtained by treating the girder as a true helicoid. The comparison will be made for a girder which subtends a horizontal angle of 180 deg and has an angle of slope along the helicoid of 30 deg. The results presented were obtained in conjunction with a study of a number of Plexiglas models. For this special case of a 180-deg girder, an analysis using the method suggested by the author yields internal forces and moments which are independent of the ratio of bending to torsional stiffness (see Fig. 2).

The helicoidal girder, fixed at the ends, under a uniform load, w, pounds per lineal foot of horizontal projection is a structure which is statically indeterminate to the sixth degree. It may be analyzed by considering the six reactions at one end (Fig. D) as redundants, and then using the principle of superposition the following equations known as the Maxwell-Mohr equations may be written.

$$\delta_{xo} + X_x \delta_{xx} + X_y \delta_{xy} + X_z \delta_{xz} + X_t \delta_{xt} + X_p \delta_{xp} + X_r \delta_{xr} = 0 \quad \ldots\ldots(1)$$

$$\delta_{yo} + X_x \delta_{yx} + X_y \delta_{yy} + X_z \delta_{yz} + X_t \delta_{yt} + X_p \delta_{yp} + X_r \delta_{yr} = 0 \quad \ldots\ldots(2)$$

$$\delta_{zo} + X_x \delta_{zx} + X_y \delta_{zy} + X_z \delta_{zz} + X_t \delta_{zt} + X_p \delta_{zp} + X_r \delta_{zr} = 0 \quad \ldots\ldots(3)$$

$$\delta_{to} + X_x \delta_{tx} + X_y \delta_{ty} + X_z \delta_{tz} + X_t \delta_{tt} + X_p \delta_{tp} + X_r \delta_{tr} = 0 \quad \ldots\ldots(4)$$

$$\delta_{po} + X_x \delta_{px} + X_y \delta_{py} + X_z \delta_{pz} + X_t \delta_{pt} + X_p \delta_{pp} + X_r \delta_{pr} = 0 \quad \ldots\ldots(5)$$

$$\delta_{ro} + X_x \delta_{rx} + X_y \delta_{ry} + X_z \delta_{rz} + X_t \delta_{rt} + X_p \delta_{rp} + X_r \delta_{rr} = 0 \quad \ldots\ldots(6)$$

In the above equations:

X_x, X_y, X_z represent the redundant forces in the direction of the x, y, and z axes, respectively

X_t, X_r, X_p represent the redundant moments acting about the x, y, and z axes, respectively

δ_{xo}, δ_{yo}, etc.—displacements due to load w with redundants equal to zero

δ_{xx}, δ_{yx}, etc.—displacements due to $X_x = 1$

δ_{xy}, δ_{yy}, etc.—displacements due to $X_y = 1$

δ_{xz}, δ_{yz}, etc.—displacements due to $X_z = 1$

δ_{xt}, δ_{yt}, etc.—displacements due to $X_t = 1$

δ_{xp}, δ_{yp}, etc.—displacements due to $X_p = 1$

δ_{xr}, δ_{yr}, etc.—displacements due to $X_r = 1$

The term displacement is understood to mean angular as well as linear displacement.

To obtain a solution for the redundants, the displacement coefficients must be evaluated. This may be done by any convenient method. For example if virtual work is used:

$$\delta_{xy} = \delta_{yx} = \int \frac{M_{rx} M_{ry}\, ds}{EI_r} + \int \frac{M_{sx} M_{sy}\, ds}{EI_s} + \int \frac{T_x T_y\, ds}{GJ_t}$$

M_{rx}, M_{sx}, T_x = bending and torsional moments in girder due to $X_x = 1$
M_{ry}, M_{sy}, T_y = bending and torsional moments in girder due to $X_y = 1$
EI_r, EI_s = bending stiffness about r and s axes, respectively (see Fig. E)
GJ_t = torsional stiffness, about t axis (see Fig. E)

After the displacement coefficients have been evaluated they may be substituted into Eq. (1) through (6). The redundants are found by solving these simultaneous equations. The procedure may be simplified considerably in the special case of the above symmetrical loading condition by selecting the redundants at midspan, since the number of unknown redundants reduces to two. The approach, however, is essentially the same.

To study the effect of various ratios of stiffnesses, four girders of rectangular cross section having different width to depth ratios were analyzed. The nominal width to depth ratios were 1:1, 4:1, 8:1, and 16:1. The ratio of E/G was 2.85 for the Plexiglas material.

The results of the analyses are shown in Table A. As noted earlier, if the structure is analyzed as a bow girder the redundants are independent of the stiffness ratios. These values are also shown in Table A for comparison.

The results shown in Table A for the helicoidal girder analyses are plotted in Fig. F, which indicates the variation in end reactions with increasing width-depth ratio of cross section. It is seen that while the end reactions are independent of the width-depth ratio in the bow girder analysis, there is a marked change in the reactions when the structure is analyzed as a true helicoid. The major difference between the two analyses is that the bow girder analysis cannot include the effect of the couple produce by the reaction forces

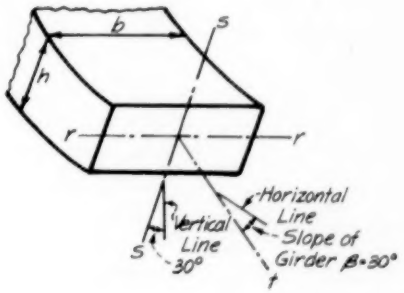


Fig. E—Bending and torsional axes indicated on section cut normal to longitudinal axis of girder

TABLE A—REACTIONS AT BOTTOM END OF HELICOIDAL GIRDER UNDER UNIFORM LOAD, w, LB PER LINEAL FT OF HORIZONTAL PROJECTION

Horizontal angle = 180 deg. Angle of slope = 30 deg. Rectangular cross section. $E/G = 2.85$

	Nominal b/h	Actual b	Actual h	$\frac{X_x}{wR}$	$\frac{X_y}{wR}$	$\frac{X_z}{wR}$	$\frac{X_t}{wR^2}$	$\frac{X_r}{wR^2}$	$\frac{X_p}{wR^2}$
Helicoidal girder analysis	1:1	0.256	0.245	−0.268	0	1.57	0.316	−0.757	0.268
	4:1	1.032	0.244	−0.857	0	1.57	0.528	−0.223	0.857
	8:1	1.987	0.244	−0.974	0	1.57	0.574	−0.117	0.974
	16:1	4.000	0.249	−1.013	0	1.57	0.589	−0.0813	1.013
Bow girder analysis	All values			—	—	1.57	0.298	−1.000	—

in the x direction at each end of the girder. This couple forms a major part of the load resisting capacity of the girder. As the width of the girder is increased more and more of the load is resisted by this couple and less and less is resisted by the moments X_r about the y axis. In the bow girder analysis all of the moment about the y axis must be resisted by the moments X_r.

As might be expected the helicoidal girder is a much stronger structure than is indicated by the bow girder analysis. To form some basis for comparison the moment diagrams for bending moments about the r and s axes and torsional moments about the t axis are plotted in Fig. G, H, and I. These moments are obtained by simple statics once the redundants are known. The following equations may be used for this purpose, using the notation shown in Fig. D and E.

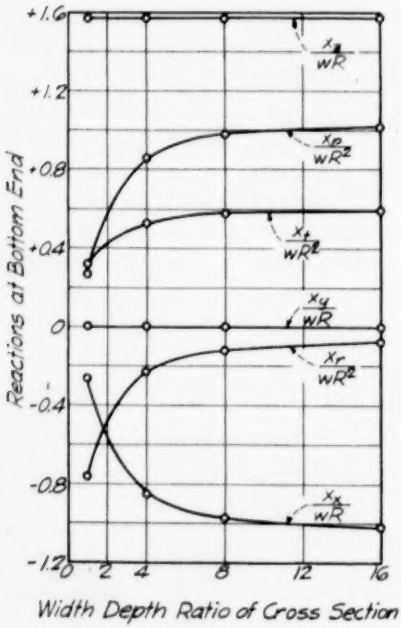


Fig. F—End reactions for uniformly loaded helicoidal girder

$$\frac{M_r}{wR^2} = -\left[1 - \cos\theta\right] + \frac{X_x}{R}\left[(\theta\cos\theta)\tan\beta\right] + \frac{X_y}{R}\left[(\theta\sin\theta)\tan\beta\right] + \frac{X_s}{R}\left[\sin\theta\right] + \frac{X_t}{R^2}\left[-\sin\theta\right] + \frac{X_r}{R^2}\left[\cos\theta\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$\frac{M_s}{wR^2} = -\left[(\theta - \sin\theta)\sin\beta\right] + \frac{X_x}{R}\left[(1 - \cos\theta)\cos\beta + (\theta\sin\theta)\sin\beta\tan\beta\right] + \frac{X_y}{R}\left[(-\sin\theta)\cos\beta - (\theta\cos\theta)\sin\beta\tan\beta\right] + \frac{X_s}{R}\left[(1 - \cos\theta)\sin\beta\right] + \frac{X_t}{R^2}\left[(\cos\theta)\sin\beta\right] + \frac{X_r}{R^2}\left[(\sin\theta)\sin\beta\right] + \frac{X_p}{R^2}\left[\cos\beta\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\frac{T}{wR^2} = -\left[(\theta - \sin\theta)\cos\beta\right] + \frac{X_x}{R}\left[(\cos\theta - 1)\sin\beta + (\theta\sin\theta)\sin\beta\right] + \frac{X_y}{R}\left[(-\theta\cos\theta)\sin\beta + (\sin\theta)\sin\beta\right] + \frac{X_s}{R}\left[(1 - \cos\theta)\cos\beta\right] + \frac{X_t}{R^2}\left[(\cos\theta)\cos\beta\right] + \frac{X_r}{R^2}\left[(\sin\theta)\cos\beta\right] + \frac{X_p}{R^2}\left[-\sin\beta\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

A study of Fig. G, H, and I reveals that in all cases the stresses obtained by analyzing the structure as a helicoid are much smaller than those obtained from the bow girder analysis. Furthermore, as the width-depth ratio increases, the bending moments about the r axis and the torsional moments about the t axis become smaller and smaller while the bending moments about the s axis become larger. These curves show conclusively that the helicoid has a great deal more strength than is indicated by the bow girder analysis.

The discussion up to this point has been limited to the analysis of a helicoidal girder which is assumed to be perfectly elastic and homogeneous. Furthermore, it has been assumed that the end reactions and therefore the internal forces in the structure may be obtained by analyzing an elastic line, defined by the longitudinal centroidal axis, having specified bending and torsional stiffnesses (thus neglecting any slab action that might occur in the wider girders).

The first assumption regarding elasticity and homogenity is common to all reinforced concrete structures and the lack of these properties must be accounted for in the factor of safety. The second assumption needs further

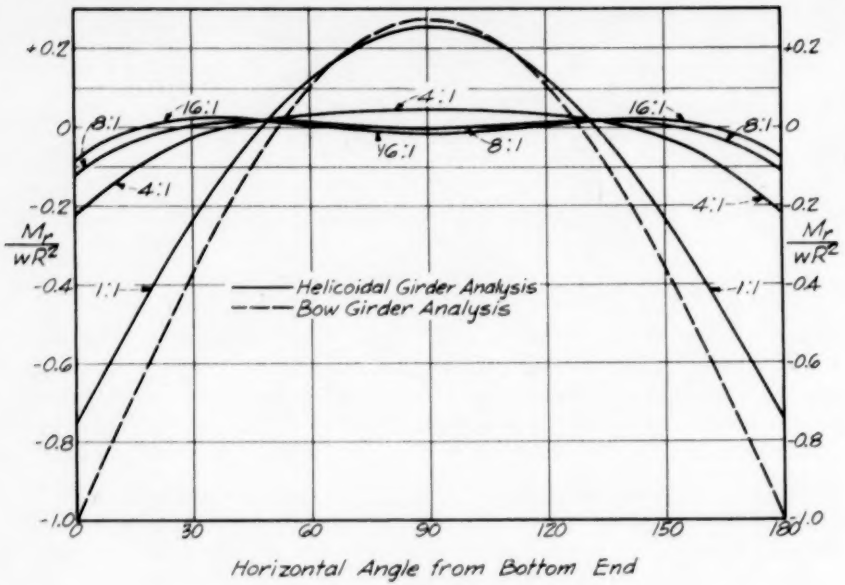


Fig. G—Bending moments about the r axis

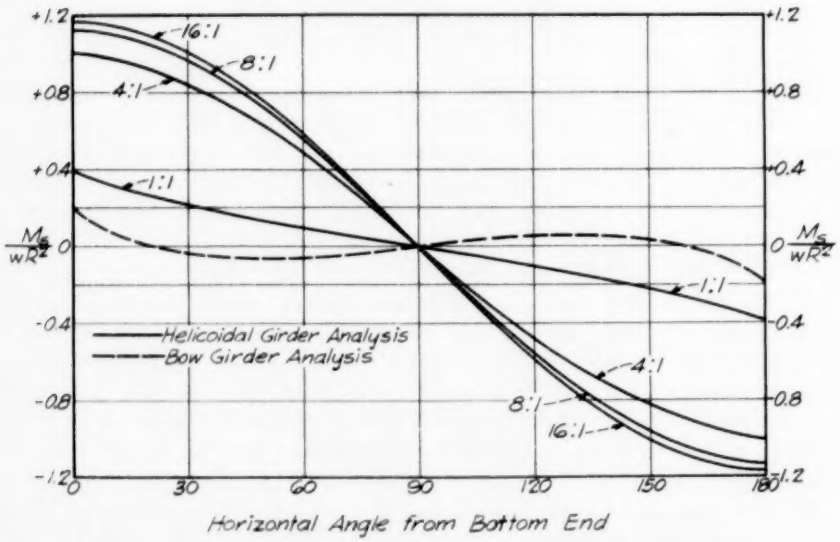


Fig. H—Bending moments about the s axis

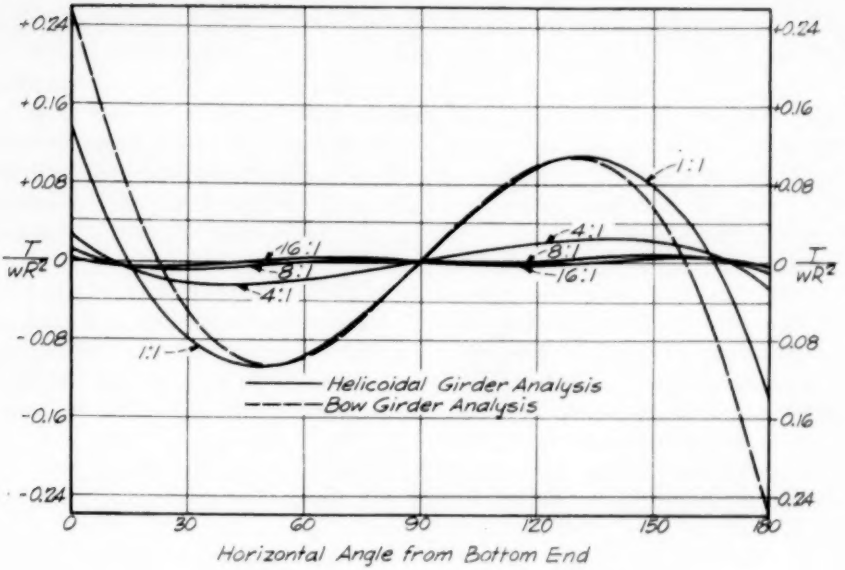


Fig. 1—Torsional moments about the *t* axis

study. Certainly for girders of small width to depth and width to radius ratios this assumption is valid, but for girders having dimensions and radii similar to those used in the illustrative example by the author this assumption is open to question. While an "exact" elasticity solution of the problem might be impractical, model studies might yield sufficient information to establish design criteria.

Finally, the whole problem of stress distribution due to the internal forces and moments needs further study. It is the feeling of the writer, however, that in a reinforced concrete structure "exact" stress formulas are not necessary, but in their place approximate methods based on engineering judgment must be used in proportioning the cross section and providing the reinforcement necessary to resist the internal forces and moments as obtained by an analysis such as has been indicated above.

AUTHOR'S CLOSURE

The author thanks the various writers who have discussed his article; he feels, and willingly acknowledges, that their contributions have added materially to the value of the paper.

The principle topic of their comment concerns the propriety of using the admittedly approximate method suggested by the author for the analysis of a helicoidal staircase. No charge is raised that the approximate method (which reduces the analysis to that of a "bow girder") is unsafe. On the contrary, Mr. Harboe offers an interesting, heuristic explanation of why the actual, three-dimensional-space stair slab has a considerable shell action which is not

taken into account when treating the problem as two-dimensional, and which provides considerable additional strength.

Professor Scordelis, in a much more extended analysis, involving the computation of stresses in a helicoidal girder subtending a horizontal angle of 180 deg and having an angle of slope along the helicoid of 30 deg, comments as follows: "A study . . . reveals that in all cases the stresses obtained by analyzing the structure as a helicoid are much smaller than those obtained from the bow girder analysis . . . the helicoid has a great deal more strength than is indicated by the bow girder analysis."

It appears that bow girder analysis has frequently been applied to helicoidal staircases designed abroad. Mr. Sanabria states: "We know of many helicoidal staircases that have been designed using this method." He goes on to say, "The method is only an approximation, but generally sufficient for the majority of cases in actual practice." Here, Mr. Sanabria expresses well the author's attitude toward the subject.

However, the author wishes to repeat his admonition that "it may be deemed advisable, in dealing with large and spectacular structures, to forego the simplifications offered by the author."

Two of the contributors, Messrs. Gedizli and Sanabria, thoughtfully provide references to more exact analyses which have appeared in foreign literature. Still another reference, furnished directly to the author through the kindness of Armin Hunziker, Lausanne, Switzerland, is the latter's "Freitragende Wendeltreppen mit starrer Einspannung und horizontale Kreisringträger," which was published in the Mar. 27, 1954, issue of *Schweizerische Bauzeitung*.

The adjective "exact" applied to an analysis should not be accepted uncritically. The various assumptions made in setting up the problem may rob the analysis of much of its validity. For example, various exact analyses that the author has seen all seem to make one assumption used by the author in his presentation of an admittedly approximate method—that the properties of the actually wide slab may be considered concentrated along its longitudinal centerline. In view of the sharp curvature involved, this assumption is questionable: for exactness, it may be preferable to analyze the stairway as a curved, warped slab rather than as a helicoidal beam. This moot question is raised by Professor Scordelis who feels that "model studies might yield sufficient information to establish design criteria." The author earnestly hopes that Professor Scordelis will soon publish an account of the results he has obtained with the Plexiglas models.

Professor Marshall voices the opinion that the author fails to give enough information regarding the design of shear reinforcement. An attempt will now be made to remedy that deficiency.

It is the author's belief that the helicoidal staircase is so decidedly an ornamental as well as functional element of construction that its cross section should be made adequate to carry the calculated torques and shears, when figured as plain concrete, without likelihood of cracking. In this, the author

is in thorough agreement with Professor Marshall who "feels that the section should be made so that it can resist the calculated torsional moment without any reinforcement."

Various sets of torsion tests on plain and reinforced specimens have shown that the presence of reinforcing steel, both longitudinal and transverse, increases the cracking strength comparatively little (roughly, some 10 percent) over that of a plain concrete member of equal cross section. However, once cracking has started, failure of the plain concrete member quickly follows, whereas the properly reinforced member still possesses a considerable reserve of strength.

Although Professor Marshall reaches the conclusion regarding the author's numerical example "that no torsional reinforcement is required," the author believes, nevertheless, that conservative design calls for torsional reinforcement. Furthermore, there are other good and sufficient reasons for providing hooping. In the first place, the hoops perform a useful function in locating and securing the longitudinal steel. Secondly, the curvature of the longitudinal reinforcement is in itself sufficient reason for the provision of hooping. For example, where one of the two inner corner bars (adjacent to the concave side of the stair slab) is in tension, the tensile force acts to straighten the bar, with resultant danger of spalling the concrete. Similarly, where one of the two outer corner bars (adjacent to the convex side of the slab) is in compression, there is a distinct possibility of the bar's bursting through the concrete cover.

To some extent, the concrete can resist such bursting or spalling effects by the development of shearing and tensile stresses. However, if we ignore the concrete's own strength, we can utilize the following simple expression for the stress in a tie bar which restrains a curved longitudinal bar:

$$\text{Tie tension} = \frac{\text{Stress in longitudinal bar} \times \text{spacing of ties}}{\text{Radius of curvature of longitudinal bar}}$$

As an example, the radius of curvature of the inner corner bars of the author's illustrative example is approximately 6 ft 2 in. = 74 in. Let us assume that the spacing of the hoop ties along the longitudinal centerline is 6 in.; then the spacing of these radially placed hoops reduces to approximately 4 in. along the innermost longitudinal bars. Either of these two corner bars can be stressed to its maximum in tension, depending on the position along the span, but not both at the same location. If the bars are, e.g., $\frac{7}{8}$-in. rounds, with a maximum stress of 20,000 psi, the maximum tension in a bar is $0.60 \times 20{,}000 = 12{,}000$ lb. Then the tension to be taken by one hoop equals $(12{,}000 \times 4)/74 = 648$ lb. If the hoops are $\frac{3}{8}$-in. rounds, the induced unit stress is $648/(2 \times 0.11) = 2950$ psi, if both horizontal legs of a hoop are considered equally effective.

Let us now consider a corner bar stressed in compression along the outer, convex side of the slab. Here, the radially placed hoops diverge from their

6-in. (assumed) spacing along the longitudinal centerline, to approximately 8 in. on center along the outermost bars. The maximum compressive unit stress in a bar may be assumed equal to $2n$ times the concrete stress at the same level. If the compressive stresses in the steel is thus found to be, say, 15,000 psi, then the load on a ⅞-in. round will be 9000 lb, and the tension to be carried by the hoop becomes $(9000 \times 8)/142 = 507$ lb, a value which is somewhat less than the 648 lb already found.

Evidently, the curvature of the main rods is a feature demanding proper attention. Another item which should receive consideration is the compression in the intermediate longitudinal rods at the bottom of the slab adjacent to the supports. Such bars constitute compression reinforcement and should be stayed against buckling, possibly by the provision of ¼- or ⅜-in. links, tying each bottom bar to its corresponding top bar. Such links would serve also as web reinforcement against direct shearing stresses.

To return now to the design of the reinforcement against torsion and shear, let us examine Professor Marshall's claim that "each method gives binders at very close spacing." In a direction parallel to each long side, the stress triangle area equals $\frac{1}{2}\ \tau_{max}\ h/2 = \tau_{max}\ h/4$ lb per in.

The torsional moment of 12,600 ft-lb used by Professor Marshall leads to a value for the maximum shearing stress of

$$\tau_{max} = \frac{12{,}600 \times 12}{72 \times 10 \times 10}\left(3 + 1.8\,\frac{10}{72}\right) = 68 \text{ psi}$$

Then, $\tau_{max}\ h/4 = 68 \times 10/4 = 170$ lb per in. A ⅜-in. diameter hoop leg, at even a conservative 16,000 psi, can carry safely 1760 lb of tension, so that the hoop spacing $s = 1760/170 = 10.3$ in. on center.

It will be noticed that the author in these computations dealing with steel reinforcement has not increased the value of the computed τ_{max} in accordance with Fig. 5. As Professor Marshall rightly points out, "Use of steel reinforcement stressed to its normal working stress implies cracks in the concrete. Under these conditions the theoretical analysis will not hold . . ." To add theoretical corrections to a questionable theoretical analysis might well be considered "confusion worse confounded."

However, a correction which may quite properly be made, is to reduce the torque vector to its component perpendicular to the normal cross section under consideration. This correction is made simply enough by multiplying the torque by the cosine of the angle made by a tangent to the longitudinal centerline of the slab with the horizontal. The other component of the torque is obtained by multiplication by the sine of the angle; this component of the torque lies in the plane of the normal cross section and produces a bending moment about the short axis of the cross section, i.e., in the strong direction. Thus, if the angle of inclination were 30 deg, the corrected torque to be used in the shear calculations would be $12{,}600 \times 0.866 = 10{,}900$ ft-lb, and the crosswise bending moment would be $12{,}600 \times 0.500 = 6300$ ft-lb.

In the direction parallel to the short sides of the cross section, the situation is complicated somewhat by the presence of parallel direct shearing stresses which, toward one of the short sides, are directly additive to the torsional shearing stresses. (The direct shearing stresses act at right angles to the

horizontally directed torsional shearing stresses along the long sides, and usually do not produce a resultant shearing stress significantly greater than the value of the torsional shearing stress alone.)

The vertical shear at the support, in the author's illustrative example, has a value of 18,150 lb. Its component in the plane of the normal cross section is 18,150 cos 30 deg (assuming again the same angle of inclination), or 15,800 lb. Then, approximately,

$$v = \frac{8}{7} \times \frac{15{,}800}{72 \times 8.5} = 30 \text{ psi}$$

This low shearing stress may surely be assigned to the concrete, leaving only the torsional shearing stress to be carried by the hoop steel. Here, the author will repeat that "the longitudinal tension steel required for bending must be proportioned to carry also the longitudinal tensile component of the diagonal tension due to torsion; the hoop steel carries only the transverse." Thus, if the required hoop steel is ⅜-in. diameter at 10.3 in. on center measured along the longitudinal centerline, then additional longitudinal tension steel is required in the amount of 0.11 sq in. for each 10.3 in. measured along the centerline of the entire length of a hoop (i.e., all four legs).

The author suggests that the foregoing analytical procedure, when applied to sections at or near the ends of the span, should be supplemented and checked in accordance with the ideas expressed in the section "Design at the Supports" (ACI JOURNAL, Oct. 1956, p. 410). Exactly what modifications are then to be made will depend largely on the judgment of the individual designer.

At this point, the author wishes to state that, regardless of computational results, he would not care to use less hooping than ⅜-in. diameter hoops at 6 in. on center; indeed, he is inclined to feel that, as a minimum, the use of ½-in. diameter hoops at the same spacing would be justified by virtue of the greater rigidity that would thus be provided to the caging of the longitudinal steel.

A different, possibly simpler, approach to the design of torsion reinforcement should be mentioned. It was recommended to the author by H. H. Bleich, and is to be found in Rudolf Saliger's *Der Stahlbetonbau* (Franz Deuticke, Vienna, 8th Edition, p. 369). Professor Saliger makes the assumption that cracking in the concrete has progressed to the extent that the steel forming a hoop is uniformly stressed throughout its entire length, and proceeds to devise a formula for the hoop steel which may be expressed as follows:

$$A_v = \frac{Ts}{2f_v A_c}$$

in which A_v = required cross-sectional area per leg of hoop
T = torsional moment
s = spacing, measured along the longitudinal centerline of slab
f_v = allowable unit stress in hoop steel

A_c = rectangular area bounded by the longitudinal centerline of a hoop = $h_c b_c$, where h_c is the distance between centerlines of upper and lower horizontal legs of hoop, and b_c is the horizontal distance across the hoop, between centerlines of legs.

The corresponding formula for the longitudinal torsion reinforcement is:

$$A_v = \frac{Tl}{2f_v A_c}$$

wherein T, f_v, and A_c have the meanings just explained
A_v = cross-sectional area of one longitudinal torsion bar
l = spacing of longitudinal torsion bars around the circumference of the member

(Incidentally, Professor Marshall gives these formulas in his paper "Experiments on Plain and Reinforced Concrete in Torsion," where he calls them Rausch's formulas.)

Paul I. Rongved, in Part 2, December, 1956, ACI JOURNAL (p. 1280), suggests that these formulas be incorporated in the ACI Building Code, together with a specification that "Torsion shear stresses alone shall not exceed those specified for shear in Section 305, and if combined with all other shearing stresses, this combination shall not exceed 150 percent of those specified for shear."

These formulas will now be applied to the 12,600 ft-lb torque already considered (without reduction of the torque by multiplication by the cosine of the stair's inclination). With ⅜-in diameter hoops, the cross-sectional area of *one* leg is 0.11 sq in., leading to a longitudinal spacing of:

$$s = \frac{2 \times 16{,}000 \times (8.375 \times 70) \times 0.11}{12{,}600 \times 12} = 13.7 \text{ in.}$$

a value which does not differ greatly from the 10.3-in. spacing previously calculated. Neither value could be considered excessively small: the author considers such spacings over-large.

The corresponding longitudinal torsion steel equals:

$$\frac{12{,}600 \times 12 \times 2\,(8.375 + 70)}{2\,(8.375 \times 70) \times 16{,}000} = 1.26 \text{ sq in. (total)}$$

or: add 0.63 sq in. to top longitudinal steel
add 0.63 sq in. to bottom longitudinal steel

Naturally, where some of the main longitudinal reinforcement is in compression, or only lightly stressed in tension, part or all of the additional longitudinal steel may prove unnecessary.

For simplicity, the author has discussed the questions of the hoop steel as torsion reinforcement and the hoop steel as ties for the curved longitudinal steel, as if they were independent problems, as indeed they are. Nevertheless,

the total unit stress in the hoop steel, from both causes, should not exceed that permitted by the governing code.

In closing, the author would like to present a few thoughts on the question of whether it is proper engineering to employ the "approximate" design instead of an "exact" analysis. Since there seems to be no question about the safety of the approximate design method, the main remaining objection would seem to be that the approximate method is ultra- or over-conservative, and therefore uneconomical.

To date, in this country, there does not seem to have been published any convenient treatment of the exact analysis, such as that provided (in German) by Fuchssteiner and Gedizli. Until this shortcoming is remedied, the expenditure of considerable time and drudgery will burden the ordinary American designer who attempts such an analysis. And what is to be gained by such an analysis? To save even as much as 4 or 5 cu yd of concrete in a stairway of moderate size, the exact analysis would have to cut the slab thickness in half. The resulting reduction in effective depth would minimize any possible savings in reinforcing steel. In formwork, usually the most important single item of cost in such a stairway, the savings would be practically negligible. Furthermore, there would remain the real problem of securing the approval of the local building department authorities, who might view the thinner slab of an exact analysis with such distrust that they would require an independent check to be made on the design, or even a loading test to be performed on the completed stairway before acceptance. In short, the savings which one might hope to obtain by means of an exact analysis may prove more apparent than real.

Discussion of a report by Subcommittee II, ACI Committee 325:

Considerations in the Selection of Slab Dimensions*

By BENGT F. FRIBERG and SUBCOMMITTEE

By BENGT F. FRIBERG†

The work of ACI Committee 325—Structural Design of Concrete Pavements for Highways and Airports—constitutes the first known attempt to collect miscellaneous published information for determination of the structural features of concrete pavements for highways and airports. Earlier subcommittee reports in the series cover structural design considerations for: (1) pavement joints (July 1956 ACI JOURNAL); (2) subgrades and subbases (August 1956 ACI JOURNAL); and (3) pavement reinforcement for crack control (October 1956 ACI JOURNAL).

The report "Considerations in the Selection of Slab Dimensions" has more direct bearing on the structural dimensions of concrete pavements than the earlier subcommittee reports. It presents design guides, many of which have been used on large mileages of highways. As exemplified by pavements with satisfactory structural performance as designed up to the present time, therefore, many of the design guides proposed in this report are supported by practical experience.

Most of the design criteria suggested in the Subcommittee II report have not heretofore been available for public discussion. Careful study of the suggested design principles is therefore appropriate before the design considerations have solidified into accepted standard design procedures of some longevity. This discussion is motivated by that need, and is no reflection upon the prodigious and valuable committee work. Rather, it is an attempt to suggest continued attention to the inadequately known and perplexing warping stress influences in concrete pavement design. Neglect of these stresses is not favorable to progressive improvement in design reliability.

BASIC DESIGN CONSIDERATIONS QUESTIONED

The safety against failure in the design consideration for corner loading of highway slabs is verified by the relative infrequency of highway corner failures. However, transverse cracks in highway pavement slabs are a more common evidence of critical stresses in present day highways, which are not

*ACI JOURNAL, Nov. 1956, *Proc.* V. 53, p. 433. Disc. 53-24 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.
†Consulting Engineer, Saint Louis, Mo.

provided against by corner load stress formulas. Transverse cracks appear to depend upon material and environmental characteristics more than upon traffic loads. Design considerations which neglect the stresses away from corners of slabs do not cover the most critical design case.

A question of principle, therefore, concerns the procedures which propose to commit a substantial part of dominant stress combinations to the realm of the safety factor. In simple terms the proposed thickness design stress considerations provide for:

(a) Highway slab design for corner traffic loads only.
(b) Airport slab design for interior traffic loads only.
(c) Exclusion from design of temperature warping stress determinations:
  (1) For plain concrete pavements, by arbitrary limitation on slab strength, based on experience with relatively thin slabs.
  (2) For reinforced concrete pavements, by providing for the transverse cracks which might occur in longer slabs because of excessively high combined warping and traffic load stresses, through steel intended to keep the cracks closed.

In accord with the above suggested principles, the only stress formulas included are for corner loads on the highway slabs and interior loads on airport slabs. The presence of temperature warping stresses is acknowledged and their consideration is suggested for plain concrete pavements but no guide to this consideration has been presented.

Combinations of load stress and warping stress apply especially for edge and interior loadings during daytime warping with maximum combined tension at the bottom of the slab directly under the load. As verified theoretically and experimentally for highway slabs, edge load stresses are of the same general magnitude as corner load stresses. This applies where joints are not doweled, and edge load stresses would be substantially higher than corner load stresses at doweled joints. Consideration of edge loads is believed to provide a more representative design condition applicable to both highway and airport slabs than the different loadings suggested for these two facilities. Inclusion of methods of determination of flexural stress for edge loading, applicable to wheel loading on day-warped slabs, is suggested.

TEMPERATURE WARPING STRESS IN DESIGN

The stresses referred to as "warping stresses" are, more correctly, warping restraint stresses. In the unloaded slab the restraint to warping can derive only from the slab's own weight and the subgrade's resistance to deflection; full restraint must exist in flat portions of long uncracked slabs some distance in from the ends and cannot exist in unloaded slabs shorter than the length necessary to develop a corresponding deadweight moment, a minimum of about 22 h for maximum day warping computed for extremely stiff subgrades up to about 35 h for subgrade k of 100 lb per cu in. With axle load on the slab, warping restraint stresses should be expected to reach full restraint under edge and interior axle loads in somewhat shorter slabs without appreci-

able relieving effect on the stress component due to wheel load. It is suggested that full warping restraint stress

$$f = E\,c\,t\,\frac{h}{2} \qquad \text{(a)}$$

should be considered in slabs 30 h long, and longer, pending inclusive research on temperature warping and its effect in slabs of different thicknesses and lengths, and on various subgrades. In the proposed consideration attention has been given to the thermal coefficient of expansion c exclusively. It is suggested that warping restraint stresses to be used in design require equally careful appraisal of E and temperature gradient t.

The modulus of elasticity E of concrete which should be used for combined stresses is undoubtedly lower than the initial tangent modulus, and E in tension also is believed to be lower than E in compression; for some aggregates it appears to be substantially lower.[1] The E value used in Fig. 1 and 2 is believed to be higher than justified for normal concrete pavement design conditions, at the age, strength, and stress value proposed for consideration.

The temperature gradient t, for which a design value of 3 deg F per in. depth is indicated, is dependent on pavement depth as well; that value would seem to apply only to thin pavements. The temperature gradient obeys physical diffusivity laws. For the most severe design condition, daytime solar temperature rise, the temperature gradient decreases with increasing depth; the warping restraint tension stress is proportionate to the difference between the mean slab temperature and the temperature at the bottom, and is correspondingly decreased. Accordingly, the maximum temperature warping stress in 12- and 15-in. slabs appears to be more nearly equivalent to a linear temperature gradient of 2.5 and 2.0 deg F per in. depth, respectively, resulting in equal full restraint stress in 12- and 15-in. slabs, only about 10 percent higher than in 9-in. slabs. No published information is known giving observed temperature gradients in the thick airport slabs.

The thermal conductivity of concrete is the basic property determining the temperature gradient, which is the direct cause of warping and warping restraint stresses. It appears to be drastically influenced by subgrade and concrete moisture content. As observed on a 7-in. experimental pavement for July warping,[2] experimentally produced exceptionally dry subgrade decreased the thermal conductivity of the concrete and resulted in unusually high temperature gradient; a fairly wet subgrade, which is a normal condition, resulted in a much lower linear maximum temperature gradient of only 2 deg F per in. depth. This observation is noteworthy as indicating low temperature warping for the conditions which would be most conducive to moisture warping. Handbook values of thermal conductivity for concrete show a range of 2 to 1, and for aggregate minerals used in concrete a range of 5 to 1, indicating the need for further research into these important relationships.

Observed temperature warping deflections of *"short"* slabs are much greater than computed for theoretical assumptions of restrained warping; the change in warping restraint stresses must be correspondingly lower. These excessive warping deflections can be explained by a rate of subgrade reaction, k, for the cyclic warping much lower than normally applicable to the rapidly imposed traffic loads, or by the existence of voids under slab ends and edges except for downward maximum daytime warping.[3] In either case short slabs, or end portions of long slabs, would be able to assume or approach the curvature of unrestrained warping. The restraint stress is a result of subgrade reaction shift toward the ends or edges; for the low rate of support near the ends, restraint stress builds up slower and reaches full restraint at greater distance from the end than theoretically proposed. While unsupported near the ends, the slabs are stressed as cantilevers, which stresses are not increased appreciably for the additional night warping of short slabs; however, these negative stresses would serve to decrease edge load stresses in longitudinal direction some distance from the ends under all conditions, except maximum day warping. The distance from the ends to sections of full restraint stress would be increased, possibly to $15h$ also for loaded slabs. For design purposes restraint stresses in shorter slabs could be assumed to increase at a parabolic rate, proportionate to the square of the distance from the end up to the distance from the end at which full restraint stress is reached; which rule would apply to maximum stress at midlength of slabs shorter than $30h$. Such tentative day warping stress determinations would give fair reality to combined stresses.

The above examples suffice to show the urgent need for exhaustive research into pavement warping and restraint stresses. Such research could be expected to remove the mystery of seemingly high combined stress in pavements with excellent performance, also to explain some design failures. Deliberate omission of warping stresses from pavement design would only serve to impede research, and appears to be a less wise alternative to their inclusion with admittedly tentative and approximate guides.

DESIGN STRESS QUESTIONED

It has been proposed to limit the stress due to wheel load to one-half of the flexural strength at mature age for the expressed purpose of avoiding fatigue. This limitation and the safety factor are not realistic, firstly, because combined stresses are higher, secondly, because fatigue stress determination would undoubtedly be based on strength at greater age which may be substantially higher. It is submitted also that impact factors should not be added to stresses based on fatigue loading. It is suggested that some 30 to 50 percent higher design stress than proposed could be applied to maximum combined load and warping stress.

Insofar as the proposed design criteria may result in pavement thicknesses which have been found adequate in actual construction, the here suggested use of higher combined stress and a higher design stress may appear

academic. This would be so if both load stresses and warping stresses were singularly related to slab thickness; however, while load stress changes in inverse proportion to h^2, warping stress changes in more nearly direct proportion to h. The combined stresses, then, change with change in thickness at a different rate than what applies to change in load stress alone. It is common to see old pavements with fortunate choice of aggregates and environmental conditions withstand traffic and full warping restraint in long slabs; some other aggregates have been observed to cause substantial increase in cracking of all but the shortest slabs. This suggests a fairly narrow division between stresses of critical magnitude and those satisfactory for long service. The safety factor which has been proposed does not appear to permit realistic design, taking all variables of load and materials into account.

THICKNESS DESIGN FOR AIRPORT PAVEMENTS QUESTIONED

Design for interior loads only has been proposed for airport slabs. Tests on model slabs[4] indicate edge load stresses to be from 1.5 to 2.0 times the stresses for interior load, and tests on full-scale slabs with doweled or keyed joints show that edge load stresses average 130 percent of interior load stresses.[5] Interior loads accordingly are not design loads determining the maximum stresses in airport slabs. The presence of warping stresses makes considerations of interior load still more unrealistic.

In the thick airport slabs warping stresses are only slightly higher than in much thinner highway slabs; furthermore, dimensions of airport slabs are frequently so limited that full warping restraint stresses cannot be reached. The real combined edge load and warping stresses can be accommodated with a safety factor consistent with loading and material limitations.

FRICTIONAL PAVEMENT STRESS

The proposed Eq. (1) is not believed applicable to design; its validity is doubtful, because it leaves out consideration of the elastic and plastic properties of concrete. If the equation were valid the cracks should quickly attain some minimum spacing at early age when the concrete strength is low, and the crack spacing should be relatively independent of other concrete properties than strength, both of which conditions are at variance with observations. Crack and joint spacing in conventional-length mature pavements is dependent on the much greater warping and load stresses and other influences, compared to which frictional stress is without significance. Frictional stress is applicable only to dimensioning of steel crossing a crack or joint, with total capacity only a fraction of the concrete's tension capacity.

REFERENCES

1. Johnson, J. W., "Relationship between Strength and Elasticity of Concrete in Tension and Compression," *Bulletin* No. 90, Iowa State College, Ames, 1926.

2. "Experimental Concrete Pavement, Topeka, Kansas," State Highway Commission of Kansas, Lawrence, 1949.

3. Friberg, Bengt F., "Pavement Research, Design, and Prestressed Concrete," *Proceedings*, Highway Research Board, V. 34, 1955, pp. 65-84.

4. Mellinger, F. M., and Carlton, P. F., "Application of Models to Design Studies of Concrete Airfield Pavements," *Proceedings*, Highway Research Board, V. 34, 1955, pp. 57-64.

5. "Final Report, Lockbourne No. 2," Corps of Engineers, Ohio River Division Laboratories, June 1950.

SUBCOMMITTEE CLOSURE

As Mr. Friberg has stated, the report of Subcommittee II on "Considerations in the Selection of Slab Dimensions" is based on principles which have been used on large mileages of concrete pavements that have given satisfactory structural performance. In the development of design theory, and in its application, it frequently is necessary to make assumptions, the correctness of which must influence the accuracy of the design. Pavement design is by no means free of unavoidable uncertainties, for even in the development of the theory, reasonable assumptions were necessary regarding such matters as radius of load distribution, the equivalent radius of resisting section, the influence of subgrade support, and others. Then in the application of the theory, reasonable assumptions must be made on several matters, such as temperature differentials, subgrade friction, and constancy of subgrade support, on which the known information is none too definite.

Regarding the basic design considerations, it was fully recognized by the committee that transverse cracking is more frequent than corner cracking, but both types of cracking are fully taken into account. It is inevitable that in a pavement slab transverse cracks will develop, due principally to warping stresses but aided also by traffic stresses as well. However, it is shown that warping stresses are critical when the slab exceeds given lengths and that they are of no serious consequence if the cracks are held together with reinforcing steel. Warping stresses are serious in unreinforced slabs and therefore plain slabs must be designed as to length to keep the warping stresses within reasonable limits. Load stresses on pavements are largest along the edges and at the corners, and therefore the case of interior slab stresses was purposely omitted, except in the case of airport runway design. The corner stress was selected for the design of the pavement thickness because if a corner forms anywhere it can cause serious breakage in the pavement, for a corner break produces a small slab subject to impact conditions and progressive deterioration.

Mr. Friberg points out that airport slabs are designed for interior traffic loads only, whereas highway slabs are designed for corner traffic loads only. He suggests that no guide has been given for the use of warping stresses in pavement design, but this matter has not been neglected in the committee report as evidenced by the inclusion of a discussion of warping by E. F. Kelly and the inclusion of curves showing temperature warping stresses in the interior and at the edge of the slab. No formula for temperature warping stresses has been given such as the well-known Westergaard formula applying to slabs of infinite length, but more particularly the formulas of Bradbury

("Reinforced Pavements," by R. D. Bradbury, and published by Wire Reinforcement Institute) and those by E. F. Kelly (*Public Roads*, July 1939, p. 96), both applying to slabs of finite dimensions. The reasons for not including formulas for warping stresses in the report are explained by the committee, which felt that certain basic knowledge was missing and therefore resort was had to the statement of practical rules for determining the length of unreinforced slabs based on past observation and experience.

The committee agrees with Mr. Friberg that a great deal of research is required to obtain all of the data which seems to be necessary for a theoretical determination of warping stresses. We are not unaware of many of the points he has brought up, but of course the writing of a design report would be long delayed if the publication of the report were dependent upon the clarification of all of the minor uncertainties.

Mr. Friberg questions the design stresses used. It is to be admitted that more data on the fatigue of concrete would be valuable, but such data as we have indicate that if the maximum bending stress exceeds half of the modulus of rupture we can expect the concrete to suffer from fatigue. The tests ("Fatigue of Concrete," *Report* No. 34-1, Illinois Division of Highways) were made on beams subjected to repeated loads at their ends with perhaps a period of recovery which was measured in seconds. The question naturally arises how this form of test compares with actual conditions in the pavement. The committee used the best information obtainable. The theory involved the selection of a wheel load which would not be repeated often enough to cause fatigue because the stress would not exceed half the modulus of rupture. The addition of an impact factor on this wheel load was intended to make for extra safety and to provide for unknown contingencies.

Regarding slabs for airport design, Mr. Friberg mentions some model tests and their indications of higher edge load stresses than interior stresses. This may well be the case, but keep in mind the larger contact areas of tires which prevent full edge or corner loadings on interior slabs. Possibly there is doubt about the criterion used by the committee for airport design being truly realistic, but the fact is that airports designed on the basis suggested seem to be acting satisfactorily.

Regarding the question of frictional pavement stress, as explained the concrete tension due to pavement friction is quite low and the "drag theory" is useful principally in making an estimate of the amount of longitudinal, transverse, and tie bar steel necessary to hold the slabs together and prevent breaking of the steel and possibly for determining the cracking of the slabs a few hours after they have hardened. Undoubtedly warping tensile stresses in concrete are much more significant than those due to subgrade friction.

It is evident that if we are ever to have a rational theory for pavement design in which all of the governing factors are accounted for in a truly scientific manner, a tremendous amount of experimental work is necessary. But this work would require years of investigation and in the meantime pavements are being built and, therefore, we must use our known theory as accurately

as possible, making as reasonable assumptions as we possibly can in the use of the theory. But at the same time we must be ever on the alert to observe the pavements in our respective localities to determine their behavior, for thus will we be able to gradually accumulate more accurate basic knowledge which will permit a more accurate application of our theory. Incidentally, concrete pavement design theory probably is as well developed as most other design theories which when applied with good judgment are giving excellent results.

Mr. Friberg deserves much commendation for reminding us of the many facts we don't know about concrete pavement design, and for pointing out specifically the paths that we need to tread in future investigations.

Discussion of a paper by Charles S. Whitney and Edward Cohen:

Guide for Ultimate Strength Design of Reinforced Concrete*

By TUNG AU, DAVID H. CHENG, C. W. GRIFFIN, JR., A. ZASLAVSKY, and AUTHORS

By TUNG AU†

The authors have indeed added some pertinent information for the design of reinforced concrete which is unavailable in the Report of the ASCE-ACI Joint Committee on Ultimate Strength Design.[1] Since the earlier works of Mr. Whitney[2,3] have been adopted as the basic tenets of the Joint Committee Report, the recommendations in this paper are authoritative as well as useful.

In the case of design of eccentrically loaded columns, the ultimate strength method provides more reserve strength for overload condition and is no doubt a safer and more satisfactory method of design. However, the Joint Committee Report leaves some unsettled questions in this regard which have been raised by the writer.[4] It is gratifying that the present paper supplies not only the additional design equations in the form of Eq. (29) and (30), but also the design charts for both square and circular columns with round cores loaded eccentrically. Aside from the fact that these charts provide time-saving aids in design, they also circumvent the need of establishing equations for limiting loads which are equally likely to cause failure in tension or in compression. In the charts of this paper, the intersections of the tension and compression curves represent the limiting loads. With many values of D_s/t and f_y, or D_s/D and f_y, as the case may be, the limiting loads thus defined are necessarily approximate, but are close enough for design purpose.

While the design charts in this paper are theoretically applicable to sections which may be controlled either by tension or compression, they are not entirely satisfactory for sections controlled by tension due to close spacings of e'/t or e'/D lines in this range. Design charts in a different form more suitable to cover the range controlled by tension have been prepared by the writer.[5] These latter charts are, in a way, similar to the diagram for rectangular members prepared by the Portland Cement Association and recommended as Fig. 6 in the Joint Committee Report.

In order to use Eq. (29) or (31) directly for the design of section controlled by tension, the limiting load must be specified for each case. For square sections with round core, the Joint Committee Report assumes that half of

*ACI JOURNAL, Nov. 1956, *Proc.* V. 53, p. 455. Disc. 53-25 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Assistant Professor of Engineering Mechanics, University of Detroit, Detroit, Mich.

the steel is effective on each side of the section and that the effective $(d - d')$ equals $0.67D_s$. Hence the limiting load for balanced design can be obtained by straightforward substitution as follows:

$$P_u \gtrless P_b = 0.36\left(\frac{90{,}000}{90{,}000 + f_y}\right)(t + 0.67\,D_s)\,t\,f_c' \quad \text{.....(a)}$$

For circular section with round core, however, the derivation of the limiting load is less obvious. First of all, the Joint Committee Report suggests that the equivalent depth of the concrete section is taken to be $0.8D$, but mentions nothing about the equivalent width. It is entirely possible to assume the equivalent width to be $0.8D$ also although such assumption gives values of P_b far below the values represented by the intersections in the design charts of this paper. Since the area of a circle is $0.785D^2$, it is perhaps more logical to surmise that D will be the approximate equivalent width so that the area t^2 for a square section is replaced by $0.8D^2$ for a circular section. Furthermore, the Joint Committee Report suggests different assumptions of effective $(d - d')$ as well as effective steel area for sections to be controlled by tension or by compression. In the case of tension failure, it is assumed that 40 percent of the total steel area is effective on each side of the section and that the effective $(d - d')$ equals to $0.75D_s$. In the case of compression failure, the assumptions remain that half of the total steel area is effective on each side of the section and that the effective $(d - d')$ equals to $0.67D_s$. This raises the question whether the effective depth of the circular section should be taken as $\frac{1}{2}\,(0.8D + 0.75D_s)$ or $\frac{1}{2}\,(0.8D + 0.67D_s)$. It has been found that the former depth gives P_u closer to the values indicated in the design charts of this paper and is perhaps a better criterion. Thus, the limiting load for a circular column with round core becomes:

$$P_u \gtrless P_b = 0.36\left(\frac{90{,}000}{90{,}000 + f_y}\right)(0.8\,D + 0.75\,D_s)\,D\,f_c' \quad \text{.....(b)}$$

The confusion due to different assumptions for effective $(d - d')$ of a circular section can probably be avoided if the design equation for section controlled by compression as represented by Eq. (32) is adjusted by letting $(d - d')$ equal to $0.75D_s$ also. Then, it leads to:

$$P_u = \frac{A_{st}\,f_y}{\dfrac{2.67\,e'}{D_s} + 1} + \frac{A_c\,f_c'}{\dfrac{9.6\,D\,e'}{(0.8\,D + 0.75\,D_s)} + 1.18} \quad \text{.....(c)}$$

This equation practically coincides with the theory advanced by Hognestad[6] and checks closely with his test results. It is hoped that the future building code requirements on ultimate strength design will provide such adjustments.

In order that Charts 4 through 15 in the paper may be used for most cases in actual practice, the upper limit of $p_t m = 1.0$ should perhaps be raised since the ACI Building Code[7] permits the use of 8 percent of steel reinforcement in rectangular tied columns subject to combined axial and bending loads as well as for square and circular columns with spiral reinforcement. Furthermore, the dimension $e = e' + d/2$ in Fig. 6 and 7 as well as in the legends for Charts 8 to 15 inclusive should probably be deleted since e is defined as the eccentricity of axial load measured from centroid of tensile steel. In the case of square or circular column with round core, the centroid of tensile steel does not lie at a distance of $d/2$ from the center of the column and the term $e = e' + d/2$ becomes misleading. Also, in a column with a round core, the diameter of the circle passing through the centers of reinforcing bars can better be denoted by D_s as in the ACI Building Code[7] rather than by the term d which has been used for more than one meaning in the Joint Committee Report. However, these points are only of minor importance and do not detract from the paper as a valuable guide for ultimate strength design of reinforced concrete.

REFERENCES

1. "Report of ASCE-ACI Joint Committee on Ultimate Strength Design," *Proceedings*, ASCE, V. 81, Oct. 1955, Paper No. 809.

2. Whitney, C. S., "Plastic Theory in Reinforced Concrete Design," *Transactions, ASCE*, V. 107, 1942, p. 251.

3. Whitney, C. S., "Application of Plastic Theory to the Design of Modern Reinforced Concrete Structures," *Journal of Boston Society of Civil Engineers*, V. 35, No. 1, Jan. 1948, p. 30.

4. Au, T., Discussion of "Simplification of Design by Ultimate Strength Procedures" by P. M. Ferguson, *Proceedings*, ASCE, V. 83, Jan. 1957, Paper No. 1156-25.

5. Au, T., "Design of Columns Controlled by Tension," unpublished manuscript.

6. Hognestad, E., "A Study of Combined Bending and Axial Load in Reinforced Concrete Members," *Bulletin* No. 399, University of Illinois Engineering Experimental Station, 1951, p. 101.

7. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-56)," ACI JOURNAL, May 1956, *Proc.*, V. 52, p. 913.

By DAVID H. CHENG*

This paper represents another welcome professional contribution by the authors. Ever since the publication of the Joint Committee Report on ultimate design in reinforced concrete, followed by its adoption as an alternative design method by the ACI Building Code, ways and means have been earnestly sought by practicing engineers to get acquainted with the new method. This timely article, together with the excellent collection of design charts seems to fulfill the current demand.

Among the many ultimate theories recommended, that of Mr. Whitney's is outstanding for its simplicity. This was achieved by introducing an artificial depth for the compression zone. If the artificial quantity so introduced satisfies all the static requirements, and at the same time is determined by existing

*Assistant Professor, City College of New York, New York, N. Y.

test data, then there is nothing improper involved. With this point cleared away, the ultimate theory for simple beams may be explained with very little difficulty.

Fig. 1 shows a typical under-reinforced beam resisting an ultimate moment. An under-reinforced beam is one in which the steel yields first. Owing to the ability of the steel to sustain large strain after the first yielding, and on the other hand, to the inability of the concrete to match with the steel strain in preserving a plane section under bending, the concrete usually is crushed before the steel breaks. However, this is primarily a steel failure, since at the impending failure of the section, the steel has yielded long before the concrete. Therefore, the ultimate capacity of the section can be computed by multiplying the yield strength of the steel by the lever arm. Eq. (1) to (6) explain the complete derivation for under-reinforced beams. Solving Eq. (6) for p, Eq. (9) is obtained.

Eq. (6) implies that the ultimate moment capacity increases with the steel ratio. Obviously, there must be an optimum steel ratio above which the capacity of the beam depends on the strength of the other partner, concrete, rather than steel. Fig. A shows a beam section conforming to such a condition where the concrete reaches its ultimate strength before the steel develops its yield strength. This section may rightfully be called over-reinforced. Taking moment about the center of reinforcement one gets:

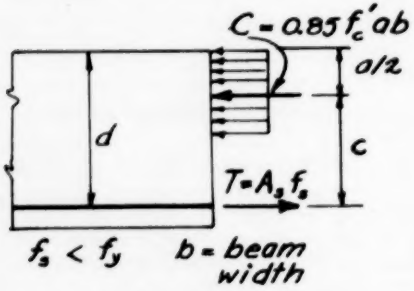


Fig. A—Over-reinforced beam

$$M_u = 0.85 f_c' abc$$
$$= 0.85 f_c' ab (d - a/2) \quad \text{(d)}$$

Simplifying, the following equation is obtained:

$$\left(\frac{a}{d}\right)^2 - 2\left(\frac{a}{d}\right) + \frac{2 M_u}{0.85 f_c' bd^2} = 0 \quad \text{(e)}$$

Solving the quadratic equation for a/d, it yields:

$$\frac{a}{d} = 1 - \sqrt{1 - \frac{2.35 M_u}{f_c' b d^2}} \quad \text{(f)}$$

it follows then:

$$\frac{c}{d} = 1 - \frac{1}{2}\left(\frac{a}{d}\right) = \frac{1}{2}\left[1 + \sqrt{1 - \frac{2.35 M_u}{f_c' b d^2}}\right] \quad \text{(g)}$$

Eq. (f) and (g) are identical with the authors' Eq. (7) and (8), and in the writer's opinion, they can only be derived for over-reinforced beams.

When the moment capacity of the beam is governed by the concrete, no useful purposes would be served by increasing steel ratio. Since the concrete strength can never be predicted as accurately as the steel, and for many other practical reasons, it would be good engineering to make all beams under-reinforced.

The critical steel ratio corresponding to the simultaneous failure of both materials was shown [Eq. (12)] to be much higher than that predicted by the existing elastic theory. Consequently, beams designed by the conventional method are all very much under-reinforced. It can be shown that a simple formula can be used with reasonable accuracy instead of Eq. (9). It is:

$$P_{req} = \frac{M_u}{f_y\, jbd^2} \qquad \text{(h)}$$

where j varies within narrow range. For average conditions, a value of 7/8 should be satisfactory. Thanks to its familiar appearance and simple form, Eq. (h) should be convenient for preliminary investigations.

The writer was thoroughly interested in the short treatment of the subject of bending about two axes. No doubt this is a topic of much practical importance, but there is very little known. The writer concedes that the general approach suggested by the authors is logical, but the final solution is still far from being complete: e.g., the nature and limit of the concrete strain on a corner, the equivalent stress block, the true location of the zero stress line, and the distribution of reinforcement in the section, all of which have direct bearing on the problem. However, more light might be shed on the subject by a systematic laboratory investigation.

Finally, the writer hopes that a set of numerical examples designed to illustrate the versatility of the charts might be forthcoming.

By C. W. GRIFFIN, JR.*

The universal bending chart for flexural members reinforced in tension only entails a rather cumbersome procedure for the design of beams and slabs. Since on a particular job the variables f_y and f_c are almost always constant it is useful to have separate design charts for different values of these variables from which direct values of A_s as a function of M_u can be selected for different values of b and d. Following is the derivation of the formula, using the rectangular stress block, from which such graphs can be plotted:

$$\text{By } \sum F = 0 \qquad\qquad \text{By } \sum M = 0$$

$$0.85\, f_c'\, ab = A_s f_y \qquad\qquad M_u = A_s f_y\,(d - a/2)$$

*Structural Engineer, Allabach & Rennis, Inc., Philadelphia, Pa.

Equating the values of a results in a quadratic equation

$$f_y^2 A_s^2 - 1.7 f_c' bd f_y A_s + 1.7 f_c' b M_u = 0$$

Solving for A_s yields

$$A_s = \frac{1.7 f_c' b d - \sqrt{(1.7 f_c' bd)^2 - 6.8 f_c' b M_u}}{2 f_y}$$

Substituting $b = 12$ in. for slab design gives A_s required for 1 ft slab width. Beam charts can also be plotted, setting $b = 10$ in., dividing beam moment by the ratio of beam width to 10 in., and then multiplying A_s for 10 in. by the same ratio of beam width to 10 in.

By A. ZASLAVSKY*

I refer to the section on slender compression members (p. 471). In authors' Eq. (38):

$$P_u' = P_u (1.6 - 0.04 l/t) \quad \ldots\ldots (38)$$

P_u is specified (also) as the carrying capacity of short columns under *combined* bending and concentric load, while under the ACI Code (A14) this load is apparently meant to be the (theoretical) ultimate *concentric* load P_o for short columns:

$$P_u' = P_o (1.6 - 0.04 l/t) \quad \ldots\ldots (A14)$$

That would imply that the carrying capacity P_u in combined bending and concentric load (which is independent of l/t) would then also apply for slender members whenever it is smaller than the value P_u' as given by Eq. (A14). Thus in the case of large eccentricities the slenderness would have no influence at all; is Eq. (38) meant to correct this situation?

By comparison, Russian codes take slenderness into account (already when $l/t > 10$) by increasing the (initial) eccentricity e' by a flexibility factor $\alpha > 1$:

$$e_{eff} = \alpha e' \quad \ldots\ldots (i)$$

This factor is given by:

$$\alpha = \frac{1}{1 - \left(\frac{\beta}{400}\right)\left(\frac{l}{t}\right)^2} \quad \ldots\ldots (j)$$

where: $\beta \cong P_u / b\, t\, f_c'$; t = depth of column in direction of bending; l = buckling length; and 400 is an empirical coefficient.

*Civil Engineer, Institute of Technology, Haifa, Israel.

In addition, the column must be checked against buckling (in the direction of the section's smaller dimension) assuming the load P_u to act concentrically, analogous to Eq. (A14).

Eq. (j) is analogous to the well-known case of an initially deflected slender bar (deflection δ_o) compressed by an axial load P. The final deflection equals:

$$\delta = \frac{\delta_o}{1 - \frac{P}{P_{cr}}}$$

where $P_{cr} = \pi^2 EA(r/l)^2$ is Euler's buckling load.

In the case of slender members the factor α is substantially greater than unity. As an example take a square column 20 x 20 in. of $l = 27$ ft unsupported length under the action of an eccentric ($e' = 12$ in.) ultimate load $P_u = 480{,}000$ lb. In this case (assuming $f_c' = 3000$ psi): $l/t = 16.2$; $\beta = 0.4$ and $\alpha = 1.36$. This means an effective eccentricity of: $e_{eff} = 1.36 \times 12$ in. $= 16.3$ in.

AUTHORS' CLOSURE

The authors wish to thank the writers whose discussions appear above as well as those who have not made formal submissions for making available their views and comments on this subject.

The purpose of this paper is to serve as a guide for the use of the ultimate strength section of the present ACI Building Code and therefore the authors have undertaken only to explain and supplement the Code, not to alter any of its provisions. In those cases where the equations in the Code and the paper differ in other than notation, the changes have been made to give what the authors believe to be the true intent of the Code. This is the case with respect to Eq. (A3) and (18), (A11) and (28), (A14) and (38). They regret the inadvertent differences in notation with respect to columns with circular cores. However, it should be noted that the definition of D_s in the code is intended to be the same as that given by the authors for d in columns with circular cores.

Mr. Cheng has verified the formulas for under-reinforced sections and has found that the straight line formulas with an average value of j approximate the required steel ratios for flexure only as computed by the ultimate strength method. However, the authors do not feel that such an approximation is necessary because c/d, which is equivalent to j, varies between 0.765 and 1.00 as limits and may be computed readily for under-reinforced flexural members as follows:

$$c/d = 1 - 0.235\, p/p_o$$

where

$$p_o = 0.4 \frac{f_c'}{f_y}$$

Although the authors agree that further study and additional tests on sections under unsymmetrical bending with and without axial load are desirable, the method outlined under "Axial Load with Bending About Both Axes," (p. 469) is believed to be conservative. It is planned to submit a complete set of design examples in the near future.

Mr. Griffin has prepared the basis for a useful design chart for a particular case, one which is often encountered.

The authors have used the design charts of this paper over a period of several years with satisfactory results, but Prof. Au's suggestions for improvements are welcome, of course. Limiting loads other than those provided by the design charts, as noted by Prof. Au, or Eq. (30) and (32), are not required for the use of Eq. (29) and (31). Eq. (b) of the discussion is not applicable to the rectangular stress block method of design.

Prof. Au's comments regarding the refinement of Eq. (32) [(A14) of the ACI Code] are interesting. However, since the only test of the accuracy of these formulas is experience and agreement with test results, such refinements are academic at this time and beyond the scope of the authors' paper which was based on present code requirements which give slightly lower capacities than the formula suggested by Prof. Au. Note that Eq. (29) to (32) are in reasonable agreement with results obtained by application of the authors' recommendations for the design of sections with the reinforcement distributed along the depth of the section.

The authors thank Prof. Au for his suggestions for improving the notation and extending the range of the design charts.

Mr. Zaslavsky's interpretation of Eq. (38) is in accordance with the authors' intention. Eq. (A14) of the Code appendix was intended to be applied in a similar manner as Eq. (17) of Chapter 2 of the Code. The axial load capacity of long columns is the lower of the two values obtained by applying Eq. (38) along each of the two axes using the column thickness t and the short-column axial load capacity P_u in each of the corresponding directions. The corresponding eccentricity in each direction is used for computing P_u. The eccentricity in any direction should not be less than that given by Paragraph A608 of the Code.

Mr. Zaslavsky also brings to the attention of English-speaking engineers the requirements of the Russian codes with respect to slender columns. Such requirements are also found in other foreign codes and are of the type described as acceptable in the final sentence under "Slender Compression Members" (p. 471). When using analytical methods for determining P_u', the engineer should exercise caution in selecting the cross section, gross or cracked, and the equivalent "modulus of elasticity" as well as the buckling length l for computing the column stiffness. It is hoped that current studies will lead to approved methods of making these selections.

Discussion of a paper by R. C. Valore, Jr.:

Insulating Concretes*

By RALPH HUNTLEY, L. E. RIVKIND, MORTON SHERMAN, HENRY TOENNIES, and AUTHOR

By RALPH HUNTLEY†

We believe JOURNAL readers should be made aware of an announcement from Riverbank Acoustical Laboratories concerning the correct interpretation of sound transmission loss test data released by us during the past several years. We have examined the graphic tabulation of sound transmission loss data in Fig. 2, p. 526, of Mr. Valore's article, looking especially at the results attributed to this laboratory. We have been able to identify about half of them in our records which go back to 1946. The other half were probably of earlier date. Many improvements have been made in the technique of measuring sound transmission loss since. From more recent data, which is not large in amount, we would judge that the numbers in the chart attributed to Riverbank are too high by about 5 or more db; a few as much as 8 db. The same estimate is made of the data in Table 5, p. 16, of the *Concrete Masonry Handbook*, AIA File No. 10-C.

We are, of course, much concerned that any data released from this laboratory be correct to the best of our knowledge. To this end we encourage wide distribution of the following announcement.

ANNOUNCEMENT FROM RIVERBANK ACOUSTICAL LABORATORIES

The completion in December, 1956, of a second reverberation room at Riverbank Acoustical Laboratories now makes it possible to measure the air-borne sound transmission loss of building materials under near ideal conditions. The new 3000-cu ft reverberation room contains the sound source, and the larger 10,100-cu ft room receives the transmitted sound. The test partition is sealed into an opening, approximately 6 x 7 ft, in the massive double-wall structure between the two reverberation rooms. In this position the test partition serves as the only significant path of sound transmission between the two rooms.

Previous to December, 1956, the large reverberation room contained the sound source and a small nonreverberant room, with certain known characteristics, received the sound. Although the technique has been thoroughly and repeatedly investigated since its adoption in the 1940's, certain assumptions about the radiated sound field had to be made which were not susceptible of experimental proof.

It appears now, with the new facility, that our earlier measurements, while relatively correct in the sense that they put materials in the proper rank order, were, on an absolute basis, about 5 db too high. This applies only to the average sound transmission loss figures in our earlier reports.

*ACI JOURNAL, Nov. 1956, *Proc.* V. 53, p. 509. Disc. 53-27 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Supervisor, Riverbank Acoustical Laboratories, Armour Research Foundation, Geneva, Ill.

It will henceforth be necessary to recognize that the sound transmission loss measurements reported by this laboratory before December, 1956, are not directly comparable with those after December, 1956. This necessity is increased since many of our sponsors have widely distributed product information based on our test results. We have two suggestions to offer:

1. If an architect has specified a certain sound transmission loss figure for a partition on the basis of his experience with partitions measured by the old technique, he should add 5 db to the average figure in reports dated December, 1956, or later. For instance, were a 35-db average transmission loss specified, a partition which we now report as having an average transmission loss of 30 db would have been reported as a 35-db partition before December, 1956, and is probably the kind of partition the architect has in mind.

2. If figures from reports previous to December, 1956, are to be used realistically or compared with later reports, 5 db should be subtracted from the average sound transmission loss.

By L. E. RIVKIND*

The grouping of insulating concretes in terms of composition serves well to describe the methods of mixing and air entrainment for the various types of aggregates when they are used. However, in line with the trend toward performance specifications, it would seem desirable to classify such concretes in terms of the required end use, namely their efficiency as thermal insulators.

Classification by performance would group concretes having oven dry densities from 15 to 40 lb per cu ft in a single category, whose prime function is thermal insulation, regardless of whether the composition be strictly cellular concrete, a lightweight-aggregate concrete, or any combination of the two. It has been shown that thermal conductivities would range from 0.5 to 1.0 Btu per hr per sq ft per deg F per in. Such concretes would find their greatest usefulness as insulating roof fills, underground pipe insulation, precast insulating slabs and block, cores for sandwich slabs, insulation of on-grade radiant heating panels, and fireproofing for steel. A minimum compressive strength of 100 psi would normally be considered adequate though this would of course increase with increasing density. Further subgroups based on strength might be used to enable a choice for specific applications.

Insulating concretes having densities from 40 to 65 lb per cu ft would have somewhat poorer insulating value but would still be quite effective while providing adequate compressive strengths for use as floor fill, roof fill and other short span, low load structural designs.

For applications where structural strength is the primary function and where it is desired to achieve some insulating value better than may be obtained with dense concretes, concretes having densities from 65 to 90 lb per cu ft should be specified. As Mr. Valore has indicated, there would be considerable overlapping of the compressive strength ranges in these three suggested insulation groupings, depending on the composition.

Of particular interest was the reported success in combining preformed foam with perlite or vermiculite aggregates and cement. In effect, this amounts to preparing cellular concrete in the usual manner and adding light-

*Technical Director, Mearl Manufacturing Corp., Roselle Park, N. J.

weight aggregate. The higher air content of such foam-plus-aggregate insulating concretes results in readily workable mixtures while simultaneously reducing the water content. For roof fills, this has the distinct advantage of reducing the "drying out" time before application of built-up roofing. As Mr. Valore has indicated, cast-in-place cellular concretes for roofs are limited to relatively flat elements. The combination with aggregates, however, results in a substantial increase in body, adequate for pitched roofs.

Recent work in our laboratory and in the field application of moist cured cellular concrete may be of interest. Using Mearlcrete foam generating equipment and foam liquid, it has been possible to further reduce water requirements for 26 lb per cu ft oven-dry neat cellular concrete to about 32 gal. per cu yd. This is equivalent to a water-cement ratio of 0.45. At this lower water-cement ratio, the fluidity of cellular concrete may be sufficiently reduced so that the required degree of "watertightness" of the supporting elements is markedly reduced.

The placement and workability of insulating foam concretes is particularly interesting. Preformed foam may be blended into a water-cement slurry in an end-loading transit mixer or a paddle type mixer. The fluid insulating cellular concrete may then be discharged into a hopper from which it may be pumped through hose lines using either pneumatic equipment or, preferably, a positive displacement pump. Proper selection of equipment makes possible the continuous placement of insulating concrete at rates as high as 20,000 sq ft of roof fill per day, from a single foam unit.

Mr. Valore's remarks relating to testing methods are particularly germane to a discussion of insulating concrete strength. In some respects, the description of these low-density materials as concretes has led to considerable misunderstanding. Frequently, compressive strength tests have been performed on saturated samples, in accord with ASTM methods applicable to dense concretes. Under these conditions, the normally low strengths are so reduced that test results show zero compressive strength. A standardized recognized series of test methods for insulating concretes are needed to give direction to architects, contractors, and test laboratories. The suggestions for testing air-dry samples with dry yielding materials such as leather or paperboard for capping have much merit. Retention for 2 days in open molds, followed by air drying to final test date more closely approaches field conditions. Since the strength of some insulating concretes varies with direction it should be specified that the compressive test load be applied in the same direction as that which will be supported in practice. It should be noted that the anisotropic character of some insulating concretes, particularly those formed by chemical reactions in which gases are evolved, may also be detected in the measurement of other properties. Due consideration should also be given in developing test procedures for evaluating these properties.

The comments on relationship between thermal conductivity and moisture content are particularly interesting. It would appear that for insulating

materials which may absorb the same total quantity of moisture, that the materials which absorb more slowly would be superior insulators under field conditions of changing humidity and moisture exposure.

The section on acoustical properties should serve well to clarify the differences between sound insulation and sound absorption.

In closing, the writer would like to take this opportunity to compliment Mr. Valore on an excellent presentation.

By MORTON SHERMAN*

In our committee work on insulating concretes, we tend to focus our attention on the aggregate primarily, rather than the entire combination of cement, aggregate, water, and air. The characteristics and physical properties mentioned in Mr. Valore's paper can all be drastically affected by the cement used, the curing or degree of hydration of the resultant concrete, and a proper or improper balance between portland cement, aggregate, air, and water components.

Of necessity, we must go through the transition stage in which individual elements in the concrete are singled out for study, before the interaction of components in combination can be thoroughly understood.

In discussing aggregate breakdown, one type of component unbalance, Mr. Valore considers primarily the substitution of air or water voids for lost aggregate volume in an attempt to maintain the concrete at a specific unit weight.

The more probable result of brittle aggregate breakdown would be the reduction of concrete volume without the addition of new volume by air or water voids. Breakdown usually occurs in the mixer during the creation of the plastic concrete; entrainment of air above and beyond that ordinarily entrained in such a mixing situation rarely occurs. While an increase in fines content could increase mix water requirements, such concrete generally remains of placeable consistency without further additions. The loss in volumetric yield is accompanied by increased density, increased strength (in relation to a situation where there is no breakdown), and reduced insulating ability.

Mr. Valore indicates the importance of sufficient air content in balance with the other components, in the prevention of bleeding and segregation problems. It should be emphasized that the stability of entrained air bubbles is as vital to quality insulating concrete as the quantity of entrained air present. This air must be of a stable nature with reference to temperature and time, or the concrete mix could undergo severe bleeding, sedimentation, and segregation effects.

As shown in Table 2, p. 517, vermiculite concretes in the 37-60 lb per cu ft density range can deliver strengths exceeding 750 psi. I question the footnote: "Not usually designed for strengths exceeding 750 psi." Vermiculite concretes of less than 60 lb per cu ft dry density can exhibit strengths in the

*Chemical Engineer, Zonolite Co., Chicago, Ill.

order of 1600 psi. As with cellular foam and perlite concretes, the limiting factor in this particular utilization would be economics.

I would like to emphasize Mr. Valore's comments on the importance of oven-dry density in insulating concretes. It is entirely true that "although the oven-dry condition may not be readily attained in a structure, it is a reproducible index of merit for control purposes." Using this index, relative strength and insulating ability can both be specified for any insulating concrete.

On p. 527, Mr. Valore makes the comment that "There is no indication that lightweight aggregates because of their 'lightness' or for any other reason, impart special sound insulation properties to concrete." I believe that damping effects are possible with certain lightweight concretes. This absorption of energy decreases the natural vibration of a monolithic wall, and results in reduced sound transmission.

Mr. Valore's paper is a needed supplement to his previous published works on cellular concrete. The paper is a thorough and conscientious reconciliation of information from many sources into an understandable compilation.

The sections on thermal conductivity and fire resistance are especially notable in that insulating concretes are particularly qualified to furnish these properties in modern construction systems. The technology of insulating concrete is of definite importance to contractors and builders, architects, and engineers. More practical and lower cost construction will be made possible through the proper use of insulating concrete.

By HENRY TOENNIES*

Both Mr. Carlson ("Lightweight Aggregates for Concrete Masonry Units," ACI JOURNAL, Nov. 1956, *Proc.* V. 53, p. 491) and Mr. Valore are to be complimented for their fine comprehensive treatment of lightweight-aggregate concrete and concrete products. However, it appears that further information is needed on the subject of sound insulation with concrete block, particularly with respect to the relationship between wall weight and transmission loss presented in Mr. Valore's paper.

A knowledge of the wall weight–transmission loss relationship has caused some architects to specify the use of solid concrete masonry units in preference to hollow. Where the choice has been between solid or hollow units of the same aggregate type (lightweight or ordinary), selection of the solid unit has resulted in wall construction of lower sound insulation value than would have resulted from use of hollow units, all other factors assumed equal.

Recent tests conducted at the Riverbank Acoustical Laboratory, under the sponsorship of the National Concrete Masonry Association, demonstrated that on a weight basis hollow concrete block are 65 percent more effective in reducing airborne sound than comparable units of solid design. Data for walls of various thicknesses, without paint or plaster, are given in Table A. As may be noted in Table A, walls constructed with hollow units gave higher transmission losses than those constructed with solid units of equal thickness.

*Development Engineer, National Concrete Masonry Association, Chicago, Ill.

TABLE A—AVERAGE SOUND TRANSMISSION LOSSES FOR UNTREATED CONCRETE MASONRY WALLS (18 TESTS)

Description of panel	Hollow units		Solid units	
	Wall weight, lb per sq ft	Transmission loss, db	Wall weight, lb per sq ft	Transmission loss, db
4-in. partitions:				
Lightweight units	18.8	34.9	24.1	29.2
Heavyweight units	24.5	42.7	36.8	35.6
6-in. partitions:				
Lightweight units	23.9	39.1	38.4	32.9

TABLE B—INCREASE IN SOUND TRANSMISSION LOSS DUE TO PAINTING CONCRETE MASONRY SURFACE (17 TESTS)

Type of paint*	RE	RE	CB	CB
Number of coats	1	2	1	2
Average increase in transmission loss, db	4.5	4.5	5.9	10.8

*RE = Resin emulsion paint.
CB = Cement base paint.

If the data shown are reduced to transmission loss in decibels per pound of wall weight, the hollow unit walls have an average insulation value of 1.8 db per lb of wall against 1.1 db per lb for the solid, a 63.5 percent difference.

The exact cause of this greater sound reduction through hollow concrete block is not known. One explanation can be found by observing the effect of painting the block surfaces (Table B).

In all cases the application of paint increased sound transmission loss, the amount of increase varying with block type and texture, and type and quantity of paint applied. Generally, the application of a cement base paint, which filled the surface pores, increased transmission loss more than a lighter resin emulsion type.

It may be reasoned that during the manufacture of hollow block, the wiping action of the core mold causes a displacement of the surface particles inside the block core. Surface voids are closed, or partially closed, much the same as the closing of voids due to troweling of a concrete slab. The effect of this void closure is a reduction in the continuous pore space through which the sound waves must travel. It is as though a light coat of cement paint were brushed upon the inside surface of the block core, resulting in a greater sound transmission loss than would occur had the concrete block been solid.

AUTHOR'S CLOSURE

The announcement from Riverbank Laboratories submitted by Mr. Huntley is both forthright and timely and brings three acoustical laboratories, Riverbank, the National Bureau of Standards, and the National Physical Lab-

oratory of Great Britain into closer agreement on the sound insulation of concrete masonry walls. It shows that large departures from the "mass law" should not be expected to occur in walls in which the surface pores of the masonry units are effectively sealed by paint or plaster. Previous assumptions that certain aggregates imparted special sound insulating properties to walls of units containing these aggregates may now be dispelled.

Mr. Toennies' analysis, it is noted, deals only with unpainted, unplastered walls of concrete block. There appears, however, to be small possibility of predicting transmission loss from weights of untreated block walls as long as surface texture remains an unmeasured variable. Rather wide variations, from "tight" to "open" texture exist in concrete block, from one plant to another. These variations may be due to a variety of causes including local preference, shape and gradation of aggregate particles, and even differences among various makes and vintages of block machines. A wall of untreated open-textured concrete block does not provide good sound insulation, whether hollow or solid. The meager pertinent data available show that heavier walls of solid block provide greater transmission loss values than those of hollow block when the surface pores of both have been effectively closed by paint or plaster. In many cases, however, the differences are not of sufficient importance to justify the use of solid block.

It may be unfortunate that sound absorption has received major emphasis as the acoustical property to be desired in a concrete masonry wall regardless of the nature of the building occupancy. The apparent values of sound absorption provided have generally not been indications of actual *absorption* of sound energy; the sound has simply been "absorbed" through a wall into the next room. Poor sound insulation or a low transmission loss has resulted. As the concrete products industry has learned from some disillusioned school architects, a partition of untreated, open-textured concrete block is a two-way acoustical channel.

The higher and higher external noise levels, due to street, rail, and air traffic, cannot be effectively diminished upon passage through a wall by absorbent treatment. The increasing use of concrete masonry for apartment party walls and for partitions between school classrooms makes it advisable to consider methods of manufacture of masonry units that will assure a good transmission loss in the wall.

Mr. Sherman has expressed the opinion that "damping" can effectively decrease sound transmission, apparently by reducing the amplitude of a wall vibrating at a resonance frequency. It is stated in National Bureau of Standards *Building Materials and Structures Report* No. 144, "Sound Insulation of Wall and Floor Constructions" (Reference 48):

> "When the sound consists primarily of a single frequency there is a possibility that the diaphragm may be in resonance with this frequency. In this case a very large part of the sound energy may be transmitted. Normally the resonance frequency of any part of a building is much lower than the frequencies of ordinary sounds, and hence this condition is not generally of importance."

A low value of Young's modulus (low stiffness) is sometimes confused with the damping capacity of a material. Some of the insulating concretes in the lowest density range (see table, top of p. 519) are quite low in secant modulus, the values being of the order of 100,000 psi. Walls of such materials would be of relatively low stiffness. We cite again Reference 48:

> " . . . There is a rather popular misconception that fiberboard and sheet lead have special properties as sound insulators. Actually, if only the sound insulating properties of the materials by themselves are considered, a sheet of steel is a slightly better sound insulator than a sheet of lead or fiberboard of the same weight per square foot because of the greater stiffness of the steel, but the difference is not usually great enough to be of practical value."

Mr. Sherman's comments on aggregate breakdown indicate a need for clarifying the point as presented in the paper. Concretes made to meet a given dry unit weight requirement, and in the mixing of which an appreciable degree of aggregate breakdown occurred, had lower strengths than otherwise similar concretes which showed no aggregate breakdown. The effect of aggregate breakdown on density was compensated for by using more than the usual requirement of air-entraining agent.

In referring to the footnote, p. 517, "Not usually designed for strength exceeding 750 psi," it should be noted that the statement refers to cellular (neat cement) concrete as well as to vermiculite aggregate concrete. There was no indication in the various references available to the author that either of these two types of concrete was ever designed for strengths over 750 psi unless sand comprised a portion of the aggregate.

The oven-dry condition described on p. 516 as a condition for density determinations and guarded hot-plate thermal conductivity tests in no way recommends oven-drying as a curing procedure or conditioning device for strength test specimens.

The author is grateful for the comments of Mr. Sherman and Mr. Rivkind on the desirability of performance specifications for insulating concretes. The main purpose in preparing the paper, however, was to examine the effect of composition upon the various relevant physical properties, rather than the preparation of specifications.

ACKNOWLEDGMENTS

The preparation of this paper was done at and supported by the National Bureau of Standards, Structural Engineering Section, Washington, D. C.

The author is indebted to many persons for supplying information used in preparing the paper. In particular, gratitude is due H. E. Robinson and R. Waterhouse of NBS who gave valuable advice concerning the sections of the paper on thermal conductivity and acoustics, respectively. Inasmuch as the author left the National Bureau of Standards shortly after preparing the manuscript, former colleague David Watstein, chief of the Structural Engineering Section of the Building Technology Division, aided in the editorial processing of the paper.

Discussion of a paper by A. M. Lount:

Prestressed Bridge Designed for Crane Load at Niagara River Weir*

By LOUIS BALOG and AUTHOR

By LOUIS BALOG†

GRID ANALYSIS

A procedure of grid analysis similar to that outlined on p. 538 was published by Zschetzsche[1] in 1893. The large number of equations made this method of computation not generally practicable at that time. Nowadays, electronic computers make such methods useful, however, not necessary in the commonly occurring cases of highway bridge design.

It is of historical interest to record that during the first decade of the century a load-distribution procedure was used extensively in the design of reinforced concrete bridges, which was in satisfactory agreement with the measured distribution of loads in actual structures and models. The design of the three-span, seven-girder Béga Canal Bridge, Temesvàr, Hungary (Fig. A and Table A), built in 1908, illustrates the application of this influence-line procedure.[2] Evaluation of the influence lines [Fig. A(e), (f), (g), and (h)] of Girders A, B, C, and D of the Béga Canal Bridge for the specified two 26,460-lb trucks and the simple beam reaction rule, give comparative design wheel loads as indicated in Table A.

TABLE A—DESIGN WHEEL LOADS, BEGA CANAL BRIDGE

Method	Girder			
	A	B	C	D
Influence line (as built)	3.11	3.35	3.33	3.18
Simple beam reaction	1.28	5.02	5.10	5.10

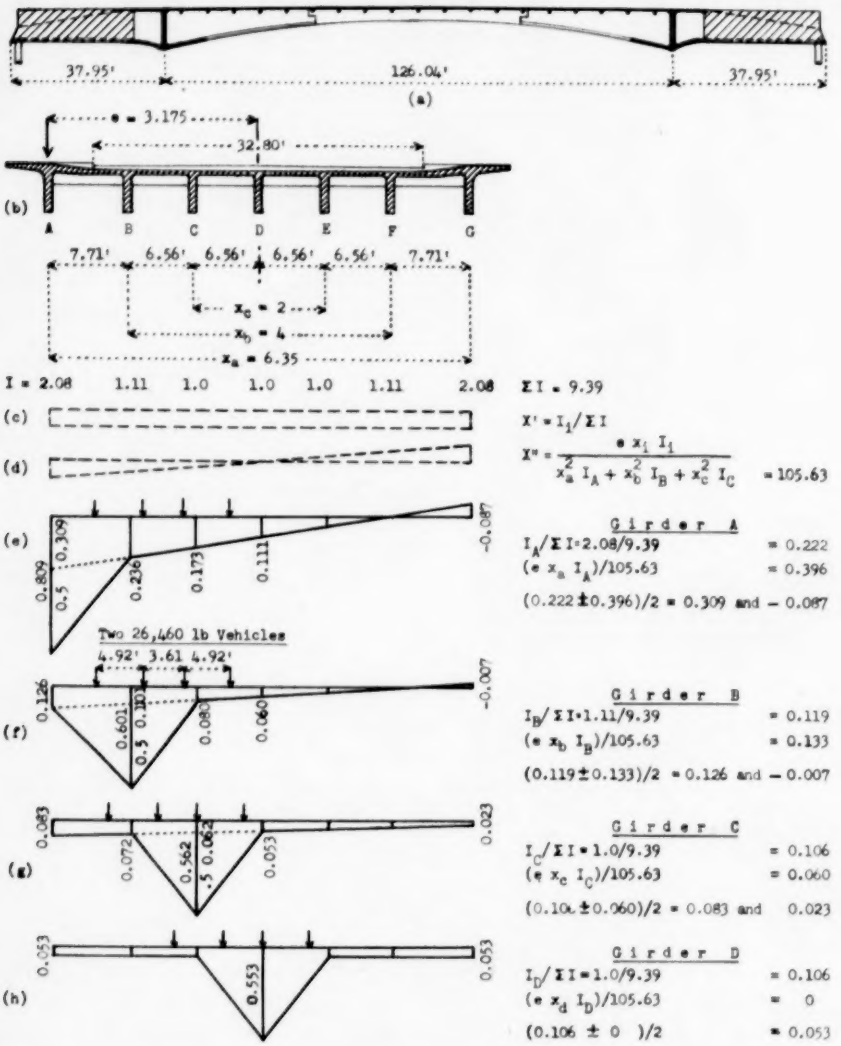
The correctness of the influence-line method was proved by measurements on actual structures. Two examples (Fig. B and C) will illustrate the relation between the computed and the measured values.

The Riet Creek Bridge (Fig. B) was investigated in 1913 by loading with a 9-ton test load. In view of the complexity of this small structure and of the simplicity of the computations, the agreement between the measured and the computed load-distribution values is satisfactory. The simple beam reaction rule, however, deviates greatly from reality.

*ACI Journal, Nov. 1956, *Proc.* V. 53, p. 533. Disc. 53-28 is a part of copyrighted Journal of the American Concrete Institute, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Consulting Engineer, Binghamton, N. Y.

The other example is Saliger's tests in 1911. Referring to Fig. C, the distribution of the load placed on Girder D among the seven girders of the grid was much more uniform at low values of P than at high values. When the magnitude of P was one-quarter of the ultimate load, the share of Girder D



$X' = I_1/\Sigma I$

$$X'' = \frac{e\, x_i\, I_1}{x_a^2\, I_A + x_b^2\, I_B + x_c^2\, I_C} = 105.63$$

Girder A

$I_A/\Sigma I = 2.08/9.39 = 0.222$

$(e\, x_a\, I_A)/105.63 = 0.396$

$(0.222 \pm 0.396)/2 = 0.309$ and -0.087

Girder B

$I_B/\Sigma I = 1.11/9.39 = 0.119$

$(e\, x_b\, I_B)/105.63 = 0.133$

$(0.119 \pm 0.133)/2 = 0.126$ and -0.007

Girder C

$I_C/\Sigma I = 1.0/9.39 = 0.106$

$(e\, x_c\, I_C)/105.63 = 0.060$

$(0.106 \pm 0.060)/2 = 0.083$ and 0.023

Girder D

$I_D/\Sigma I = 1.0/9.39 = 0.106$

$(e\, x_d\, I_D)/105.63 = 0$

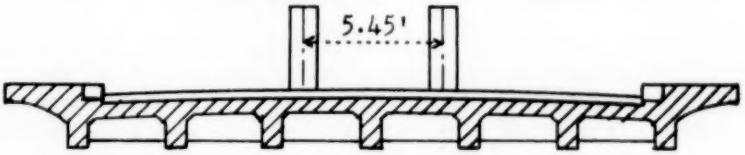
$(0.106 \pm 0)/2 = 0.053$

Fig. A—Bega Canal Bridge

Skew—53 deg. Rectangular floor framing; 5.51 in. two-way slab; crossbeams 5.9 in. wide, 7.87 in. deep, spaced 6.56 ft

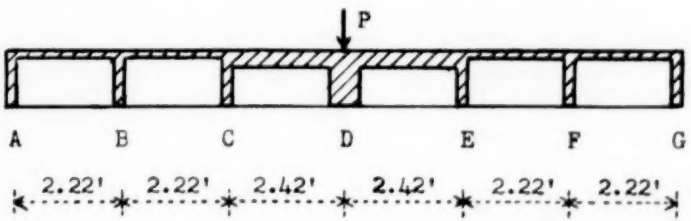
was 21 percent smaller than at the ultimate load, of which Girder D carried 62 percent. Using the four full moment inertia values of the girders in the kind of computations shown for the Béga Canal Bridge (Fig. A), the share of Girder D is 64 percent, in good agreement with the measured value at the ultimate load.

About 40 papers had appeared on load distribution in framed systems, reporting theory, measurement of actual structures, and design of new bridges, when Ostenfeld's[3] book on this subject was published in 1930. The assembled



0.50	1.10	1.80	2.00	2.00	1.10	0.50	Measured
0.65	0.66	2.20	1.98	2.20	0.66	0.65	Computed
0	0	3.19	2.62	3.19	0	0	Simp.B-R

Fig. B—Distribution of 9-ton test load on Riet Creek Bridge



0.046	0.163	0.300	0.491	for P = 17,640 lb	Measured
0.010	0.090	0.280	0.620	for P = 70,550 lb	Measured
			0.640		Computed
			1.000		Simp.B-R

Share of Girders from Load P

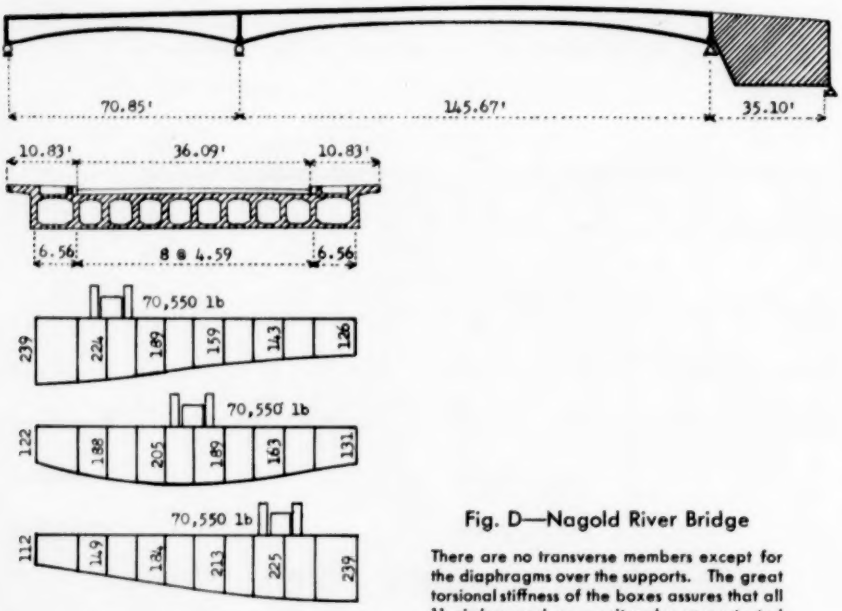
Fig. C—Saliger's 1911 tests

knowledge and the new investigations presented in this book directed attention to the great economic significance of the proper design of grids. The widespread practical application of the principles of grid design began at this time. Subsequent excessive use of grid bridges in central Europe contributed to the development of simplified computations for grids. Among these, Leonhardt's method,[4] published in 1938, neglecting torsional stiffness, has been used extensively in bridge design. The work of Enyedi[5] 10 years earlier and the more recent work of Homberg[6] give identical results.

LOAD DISTRIBUTION BY TORSIONAL RIGIDITY

Load distribution by torsional rigidity was first empirically considered. Examples of this procedure are three box-girder bridges built from 1912 to 1928 in Pforzheim, Germany. These have no other transverse members than the diaphragms over the supports. Among these the Nagold River Bridge, built in 1928, is 60.75 ft wide, has 11 girders, about an 18-in. floor slab, and a bottom slab varying in thickness from 7 in. to about 16 in. at the supports. The measured deflections of the 145.67-ft span of this continuous girder bridge are shown in Fig. D. Based on previous experience, the load distribution corresponding to these deflections was considered in the design of this bridge.[7]

Experience with numerous bridges indicated that the oldest and simplest method of load-distribution computations, published by Engesser[8] in 1889



Measured Deflections in 1/10,000 In Units

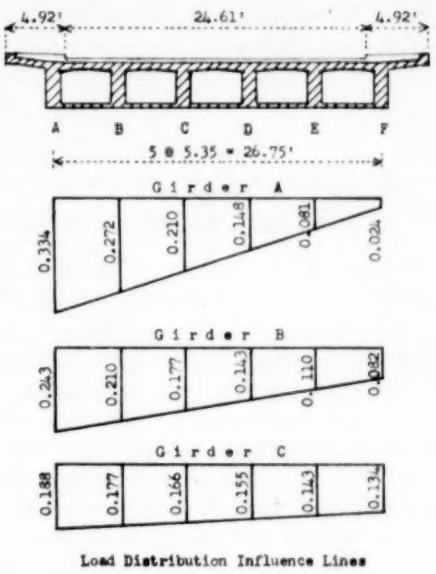
Fig. D—Nagold River Bridge

There are no transverse members except for the diaphragms over the supports. The great torsional stiffness of the boxes assures that all 11 girders work as a unit under concentrated load

Prestressed concrete continuous box girder

Distribution of H-20 Truck Loading

Method	Wheel Load on Girders		
	A	B	C
Influence Lines	0.8228	0.7054	0.6626
AASHO	0.4261	1.0700	1.0700
Difference %	48	34	38



Load Distribution Influence Lines

Fig. E—Kinzing River Bridge

and 1892, when extended by the consideration of the torsional rigidity of the grid, gave for design purposes satisfactory results. The prestressed concrete continuous box-girder bridge built at Steinach, Germany, in 1955, is an example (Fig. E).[9] This method was applied also in the design of the four-girder, prestressed concrete bridge, 918.63 ft long, 40.19 ft wide, built at Northeim, Germany, in 1955. It has seven continuous spans, each 131.23 ft long with three equally spaced intermediate diaphragms in each span. The transverse spacing of the girders is in the ratio of 1 : 1.12 : 1; the vertical and torsional rigidities of the outer and inner girders are different. Nevertheless, this simple method of load distribution gave entirely satisfactory results.[10]

The load-distribution diagrams of the 14-girder bridge on p. 538 (Fig. 5—Load distribution in a torsional system) can also be satisfactorily approxi-

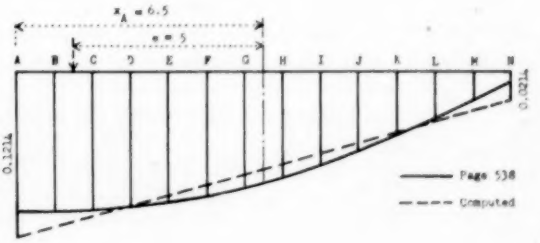


Fig. F—Approximation of actual load-distribution diagram by assuming a rigid diaphragm

mated for highway loads by Engesser's equation extended by the torsion coefficient of Schöttgen,[11] as follows (Fig. F):

$$t_A \text{ and } t_N = \frac{1}{14} \pm \frac{x_A \, 8}{\sum x^2} \overline{\tau}$$

where $\overline{\tau} = 1/[1 + (14/12 \times G/E \times I_g{}'/I_g \times l^2/\sum x^2)] = 0.35$

t_A and $t_N = 0.0714 \pm (0.1429)\,0.35 = 0.1214$ and 0.0214, respectively

In case of rigid diaphragm the load-distribution diagrams are straight lines defined by the end ordinates t_A and t_N. These can be computed for all girders by introducing the corresponding e value. The evaluation of these influence lines for highway truck loadings give $U^{(4)}$ and $U^{(5)}$ loads which are practically identical with those obtained from the published diagrams. The moments and shears produced by these loads must be computed with due consideration of the actual diaphragm spacing.[10]

Load distribution in slab-framed, two-girder bridges without diaphragms is influenced by the slab thickness and also by the width of the stem of the girders and by the torsional rigidity of the end floor beams in short spans. Table B contains the dimensional properties of Bridge 1, discussed by the writer,[10] and of Bridge 2, rigorously analyzed by Bechert.[12]

Table C shows that the 67 percent wider girder stems of Bridge 2 have a noticeably advantageous effect on the load distribution; likewise, the large torsional rigidity of the end floor beams of short Bridge 2 increases the center moment influence ordinate of unloaded Girder B by 10.4 percent.

The approximate load-distribution ordinates are:[10]

$$t_A \text{ and } t_B = 0.5 \pm \frac{3}{6 + \tau} \qquad (1)$$

$$\text{where } \tau = (l/2a)^3 \, G I_g{}'/E I_g \qquad (2)$$

Taking $a = a - b'$ in Eq. (2), Eq. (1) then yields 0.7483 and 0.2517 (the AASHO specifies 1.0 and 0.0). The ordinates of this diagram, which can be obtained in a few minutes, are compared with the rigorous values of Bechert in Table D.

The evaluation of these two influence diagrams for any of the AASHO truck loadings gives practically identical midspan girder moments.

If the torsional grid stiffness is neglected, load-distribution diagrams can be conveniently computed by simplified methods which are similar to that published by Enyedi,[5] Leonhardt,[4] or Homberg.[6] The practical identity of

TABLE B—PROPERTIES OF BRIDGES 1 AND 2

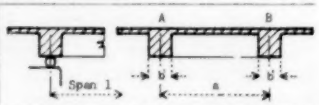
$G = 0.5E$	l, ft	a, ft	b', ft	I_g, ft⁴	$I_g{}'$, ft⁴	I_c, ft⁴/ft	I'_{FB}, ft⁴	
Bridge 1	90.55	17.39	1.97	17.00	10.08	0.0795	—	
Bridge 2	65.62	16.40	3.28	29.78	23.17	0.0450	23.17	

TABLE C—EFFECT OF WIDTH OF GIRDER STEM ON LOAD DISTRIBUTION

	$\frac{a^2}{d^2}\frac{E I_g}{G I_e}(n\pi)^2$ (1)	$\frac{E I_g}{E I_c}\frac{a^3}{24 l^4}(n\pi)^4$ (2)		$\frac{1}{[1 + (1) + (2)]\pi^2}$		Moment influence line ordinate of Girder B		Difference, percent
		$a = a - b'$	$a = a$	$a = a - b'$	$a = a$	$a = a - b'$	$a = a$	
Bridge 1	$0.308\ n^2$	$0.047\ n^4$	$0.068\ n^4$	0.0748	0.0736	6.77	6.67	1.47
Bridge 2	$0.396\ n^2$	$0.321\ n^4$	$0.474\ n^4$	0.0590	0.0542	3.87	3.54	8.30

TABLE D—INFLUENCE-LINE ORDINATES OF MIDSPAN GIRDER MOMENT

Method	0.5 l		0.375 l		0.25 l		0.125 l	
	A	B	A	B	A	B	A	B
Bechert	12.08	4.32	11.33	4.05	9.17	3.13	5.46	1.72
Eq. (1) [$a = a - b'$]	12.27	4.13	11.56	3.82	9.38	2.92	5.59	1.58

the computed diagrams with those published on p. 537 is indicated in Fig. G.

Take the example of a five-girder bridge on p. 537, Fig. 4:

Assume $z = 20$ $\quad 7 + 34\,z + 5\,z^2 = 2687$ $\quad 4 + 5\,z = 104$

Girder B: t_A and $t_E = (2.5\,z + z^2)/2687 \pm z/104 = 0.3598$ and -0.0248, respectively

t_B and $t_D = (3.5 + 9\,z + z^2)/2687 \pm (2 + 0.5\,z)/104 = 0.3325$ and 0.1018, respectively

Girder C: t_A and $t_E = (3\,z + z^2)/2687 = 0.1265$

t_B and $t_D = (11\,z + z^2)/2687 = 0.2307$ = also t_C for Girder B

$t_C = (7 + 18\,z + z^2)/2687 = 0.2855$

The above figures show that the published diagrams are closely approximated by the simple computations. Evaluation of the published and computed load-distribution diagrams for highway truck loadings gives for design purposes practically identical results.

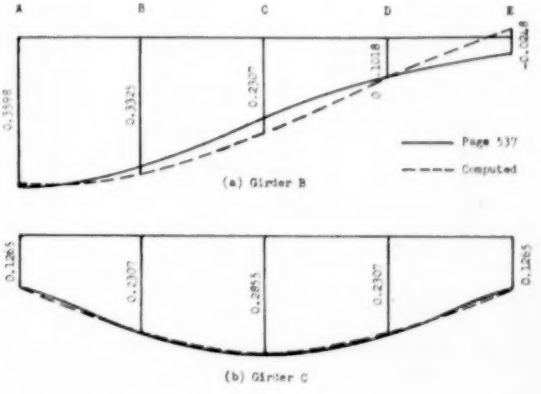


Fig. G—Approximation of actual load-distribution diagrams by simplified computations

It is regrettable that no load-distribution diagrams and moment and shear values obtained by the computations outlined on pp. 538-539 were published for the 70-ft span, six-girder grid, 24 ft wide with intermediate diaphragms spaced 8.75 ft. The comparison of these values with those obtained by simplified computations in general use would have been of interest. Measurements on a large number of actual bridges indicated good agreement with values computed by the simplified methods. Generally, the actual distribution was better than computed.

VALIDITY OF COMPUTATIONS

The validity of the computations is influenced by the accuracy of the vertical and torsional rigidity values and their ratios used. In case of steel grids these stiffnesses and their ratios vary according to the connecting media of the assemblies and of the character of the connection details. The rigidity characteristics of reinforced concrete grids are dependent on the cracks in the members and on the extent the reinforcement hinders the deformations. The stiffness of the cracked girders is much smaller than of the full sections and if the transverse elements remain free from the same type of cracks the stiffness ratios will be different from those used in the computations. The writer suggested 12 years ago to the Bureau of Public Roads a research program establishing the rigidity characteristics of various types of actual grid bridge structures and the extension of specifications so as to conform to the actual behavior of bridges.

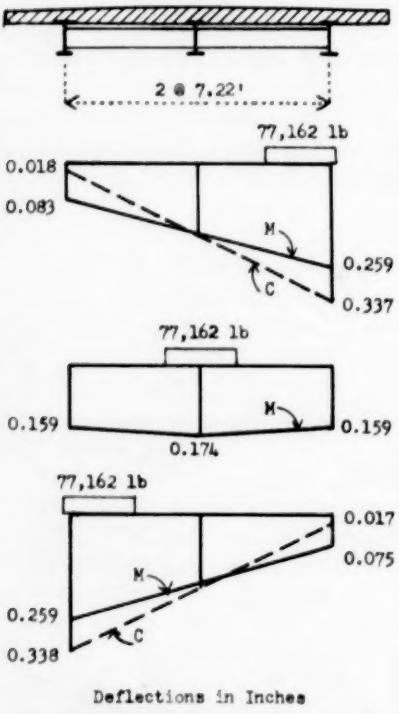


Fig. H—Computed (C) and measured (M) deflections of composite bridge

Measurements on composite bridges, steel girders with heavy anchored floor slabs, invariably indicated torsional rigidity which influenced load distribution; Fig. H shows an example. The measured deflections of the five 40-ft spans of the Oberhausen Bridge, built in Germany in 1949, indicated 20 percent more uniform distribution of the test loads between the girders than the computations neglecting the torsional rigidity of the spans.

SLAB THICKNESS IN COMPOSITE CONSTRUCTION

The statement on p. 535 that, "In composite . . . construction . . . a deck thickness of 7 in. can be considered critical," is in agreement with the German composite highway bridge specifications.[13] These require for first class highway bridges a minimum slab thickness of 7.87 in., for concrete cube strengths from 4270 to 5690 psi, and 7.09 in. for concrete cube strengths from 6400 to 8530 psi. The use of even somewhat thicker slabs, or the provision of flat haunches, or the transverse prestressing of the slab is recommended.

REFERENCES

1. Zschetzsche, A., "Theorie lastverteilender Querverbände," *Zeitschrift des oesterreichischen Ingenieur- und Architekten- Vereins* (Vienna), V. 54, 1893, pp. 553-562, 572-579, 588-593, 607-611.

2. Michailich, Győző, *Vasbetonszerkezetek*, V. I, Németh, Budapest, 1922, pp. 317-327.

3. Ostenfeld, Christen, *Lastverteilende Querverbande*, Gjellerup, Copenhagen, 1930.

4. Leonhardt, F., "Die vereinfachte Berechnung zweiseitig gelagerter Trägerroste," *Die Bautechnik* (Berlin), V. 13, 1938, pp. 535-, 553-, V. 14, 1939, pp. 252-, 419-. Latest edition: Leonhardt and Andrä, *Die vereinfachte Tragerrostberechnung*, Hoffmann, Stuttgart, 1950.

5. Enyedi, Béla, "Die Verteilung von Einzelkräften auf nebeneinanderliegender Träger," *Beton und Eisen* (Berlin), V. 27, 1928, pp. 352-355.

6. Homberg, H., and Weinmeister, J., *Einflussflachen fur Kreuzwerke*, Springer, Berlin, 1956.

7. Seibel, L., "Weitgespannte Eisenbetonbalkenbrücken in Pforzheim," *Beton und Eisen* (Berlin), V. 30, 1931, pp. 25-31, 41-46.

8. Engesser, F., *Die Zusatzkrafte und Nebenspannungen eiserner Fachwerkbrucken*, Springer, Berlin, 1892, pp. 83ff.

9. Zschiedrich, E., "Die neue Kinzingbrücke bei Steinach," *Die Bautechnik* (Berlin), V. 33, 1956, pp. 225-233.

10. Balog, Louis, "Grid Bridge Design," *Civil Engineering*, V. 27, No. 1, Jan. 1957, pp. 40-45.

11. Schöttgen, J., "Einfluss der Verdrehungssteifigkeit der Hauptträger auf die Lastverteilung beim Trägerrost nach Rechnung und Versuch," *Bautechnik-Archiv* (Berlin), V. 1, 1947, pp. 40ff.

12. Bechert, Heinrich, "Einflussflächen zweiseitiger Plattenbalken," *Beton und Stahlbetonbau* (Berlin), V. 52, 1957, pp. 17-21.

13. *DIN 1078*, Verbundträger-Strassenbrücken, paragraph 6.11, Table 1.

AUTHOR'S CLOSURE

The author thanks Mr. Balog for his interesting discussion and his review of the empirical methods commonly used for the calculation of bridge grids. The author regrets that, since the nature of the article was more concerned with a single practical application, space and time did not permit a more detailed discussion of the background for the type of grid analysis used. In a paper given at Jackson, Miss., to the American Society of Civil Engineers, the question of the load distribution between girders and, in particular, the comparison between an exact elastic analysis and the approximate empirical analyses developed by various authors is discussed.

Also, in this paper a load-distribution test carried out on the structure at Niagara was discussed. Fig. I and J are from this paper. Fig. I shows a

comparison between the maximum moment envelopes on an individual girder for a six-girder structure over the Big East River near Huntsville, Ont., with a span of 120 ft, giving a comparison between the elastic analysis and an approximate elastic analysis by Guyon. Those methods neglect torsion. It is interesting to note that, while the moment values would appear at first sight to be in approximately the same range, there is actually a significant reversal of results, particularly with reference to the inner girders.

It has been our experience in the application of the empirical methods that the methods mentioned by Mr. Balog are not sufficiently sensitive to permit close design. They can be used as a design basis provided that sufficient allowance is made for inaccuracies, although they do represent a definite improvement over other arbitrary empirical methods.

It will be noted that the transverse load distribution computed by the elastic method for the Niagara bridge is substantially more accurate than the method using the approximate elastic analysis neglecting torsion (Fig. J). It is also more accurate than the corrected approximate elastic analysis as suggested by Massonet, taking into consideration torsion (Fig. J).

The author is not too convinced that we can, at this stage, rely too heavily upon torsional resistance of single concrete girders in bridge structures. We have little information as to the long-term effects of creep, cracks, and weathering on the eventual torsional resistance. We must remember that torsional stiffness means corresponding torsional stresses. These stresses must be considered in the design and provided for.

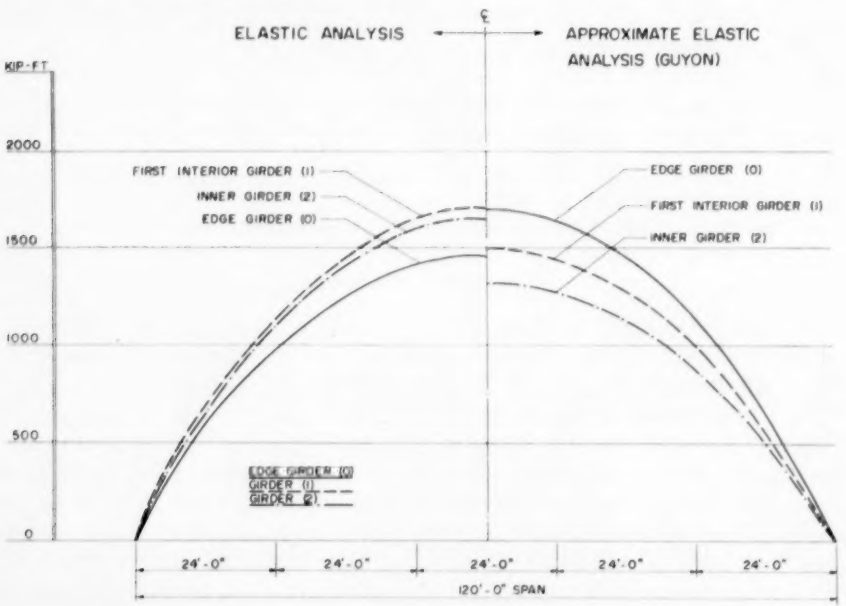
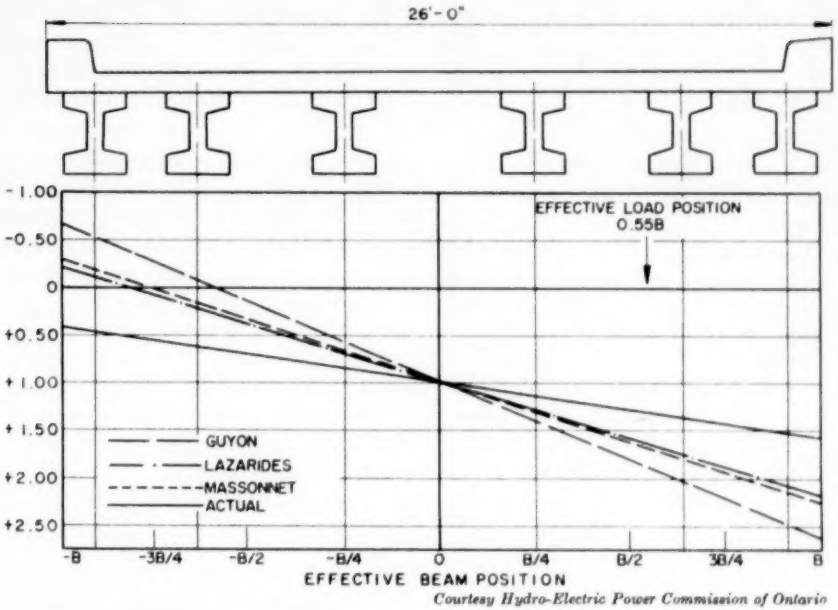


Fig. I—Maximum bending moment envelope due to live load



Courtesy Hydro-Electric Power Commission of Ontario

Fig. J—Distribution coefficients for load No. 2, Niagara remedial works

This brings into focus one of the most interesting aspects of this whole question, namely, the difference between the rational and the empirical approach. The rational approach makes initial assumptions which are as close as possible to the actual conditions existing and proceeds from that point on the basis of exact calculations and computations to derive the actual stresses which exist in structure. Any variations from measured stresses as compared to computed stresses in such methods are due entirely to errors in the initial assumptions.

In the empirical approach, a double assumption is made. Not only are initial assumptions made but the calculation of the behavior of the structure based on those initial assumptions is also itself based on assumptions. There is, therefore, in the empirical approach a double source of error. This does not prevent valuable contributions by means of empirical methods. However, it does mean that the extension of empirical methods to fields beyond which they were strictly intended is questionable, i.e., variable girder spacings.

The rational approach is far more generally applicable. The author regrets that he cannot agree with Mr. Balog's statement that the use of such methods are not necessary. It is the author's contention that valid rational approach is always preferable to a corresponding empirical approach, especially when the difference in cost is slight. It is true that a large number of successful designs have been based upon the empirical approaches mentioned by Mr. Balog, and I feel that we are much indebted to Mr. Balog for

his interesting and intensive review of the literature on the subject. This literature, as is pointed out by Mr. Balog, certainly shows that ever since even as early as 1893, engineers charged with the responsibility of designing bridge structures, have been convinced of the importance and desirability of adding diaphragms to bridge structures to obtain the most efficient use of the material at their disposal. However, we feel that the empirical methods suggested by the various authors quoted by Mr. Balog, bear the same relationship to rational design methods as the empirical expressions $WL^2/10$, $WL^2/24$, and $WL^2/12$ bear to moment distribution and other rational methods of analysis commonly used in the computation of moments in continuous structures. There is a place for each in our assortment of available design tools.

Discussion of a paper by B. J. Butcher and H. J. Hopkins:

Particle Interference in Concrete Mixes*

By T. C. POWERS, FRITZ SCHWANDA, H. N. WALSH, and AUTHORS

By T. C. POWERS†

Messrs. Butcher and Hopkins should be thanked for their paper. It throws needed light on the intriguing but oversimplified Weymouth theory of particle interference. Once pointed out, it seems clear that particle interference does not occur as pictured by Weymouth unless the mix contains only two sizes of particles. When various sizes are present in a fully compacted mix, all the small particles find places among all the larger particles; when consolidation is incomplete, the bridging of particles that prevents full consolidation may involve many different sizes around each void.

The method of analysis indicated in Table 1 reminds one of a concept associated with the use of Abrams' fineness modulus, which concept can be expressed as follows: If one removes aggregate from a concrete mix in progressive steps by "wet sieving," using successively smaller sieves, the fineness modulus of the aggregate remaining in the mix after each operation must not exceed the "maximum permissible fineness modulus" for the remaining maximum size of aggregate and cement content. (The maximum permissible values of fineness moduli were established empirically by Abrams and his associates.)

This paper supports the contention that an adequate theory of mixes must deal with all particle sizes at once, just as is done when making trial mixes. The authors, however, do not deal with *all* the particles, and that is to be regretted. They chose to apply their ingenious method to *dry* mixtures of cement and aggregate. The reasoning that led them to include the cement along with sand and coarse aggregate seems to offer no logical escape from the necessity of including the still smaller and all important particles, the water molecules. Calculations for d^3 of column 4, Table 1, show that in the dry system, relative average size is determined by the size of the cement particles, the weighted size of the sand particles being negligible. If the computation had included water molecules, the weighted size of the cement particles would also have been negligible.

Of course, it seems absurd to talk about a system of particles comprising coarse aggregate, sand, cement, and water molecules. Water molecules do

*ACI JOURNAL, Nov. 1956, *Proc.* V. 53, p. 545. Disc. 53-29 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Manager, Basic Research Section, Portland Cement Association, Chicago, Ill.

not "pack" like ordinary particles. They exert forces of attraction and repulsion on each other, and because of this they remain separated by a fixed average distance.* The average distance being larger than the molecules themselves, it is possible for the aggregation to be mobile, i.e., fluid, but, because of interparticle forces, it is also cohesive; it hangs together. Nevertheless, water molecules do occupy space in a concrete mix, and in any workable mix they occupy spaces between the particles of cement and between particles of aggregate at places where the particles would normally be in contact in a dry mix. In other words, water molecules prevent the aggregate and cement particles from packing as they do in a dry mix. Such separation is not related to the size of the water molecules but to the consistency of the molecular aggregation, water. It is obviously not necessary to deal with individual molecules of water to determine what space they will require in a concrete mix; it would be futile to try to do so.

If, because of the effects of interparticle forces (intermolecular forces), we do not include water molecules in a grading analysis, we should for the same reason exclude cement and other particles of like size from the grading analysis. Fine particles in water or, rather, in a dilute solution, also do not "pack" as do coarser particles; they occupy all the water-filled space provided for them. In this medium, they, like water molecules, exhibit effects of interparticle attraction and repulsion, and the system, particles-plus-water, has a certain consistency. The function of paste-making particles in water-filled space is more like the function of water molecules in water-filled space than it is like the function of sand particles in an aggregate. Thus, I can agree with the statement on p. 551, ". . . it is clear that all solids should be treated in the examination," but I cannot agree to the procedure that treats them all alike, and ignores the water.

Paraphrasing the last sentence in the first paragraph of this discussion, we can say that in concrete the small particles of aggregate, together with the paste, find places among all the coarser particles. Hindrance to complete consolidation comes not only from particle interference among aggregate particles, but also from the consistency of the paste (its stiffness), and capillary forces around air bubbles in the paste. Experiments have shown that consistency of the paste is the important thing and capillary force cannot be ignored; the size, shape, and gradation of the particles in the paste is a secondary factor.

Because of these facts, Weymouth's idea, and the present outgrowth of it, have intrinsic limitations; they do not deal with the most important factors that determine the compactability of a concrete mix. Although we can admire the skill and ingenuity of the analysis, we must look in other directions for an adequate theory of mixes.

* * *

Incidentally, this reader was confused by the discussion under "Interpretation of Results." The first sentence gives the new criterion for particle

*Temperature constant.

interference, but the following sentences and Table 2 seem to be based on the original Weymouth theory and not Eq. (15a). In fact, the figures in the upper part of the table appear to have been taken from one of Weymouth's original papers; I also used them in a paper written in 1936. It is hoped the authors will clarify this part of the paper by further discussion.

By FRITZ SCHWANDA*

Gradation of the aggregates not only affects the workability and quality of a concrete, but it is a decisive factor in the economy of a mix. Extensive labor, experimental work, and funds have therefore been spent on exploration of the problem of aggregate gradation since Fuller's pioneer investigations at the beginning of our century.

The solution of that problem is possible only with the aid of an exact knowledge of the mechanism of particle interference, a phenomenon which Weymouth was the first to treat in a scientific way, though Furnas had paid proper regard to it in his method of design of gap gradings 2 years before Weymouth's publication (see Furnas, "Grading Aggregates," *Industrial and Engineering Chemistry*, V. 23, 1931, p. 1052).

We are indebted to Messrs. Butcher and Hopkins for having resumed a question of such consequence. The authors' method helps us a great deal to understand the nature of particle interference. Yet multitudes have their own laws, which do not always comply with the laws which we can deduce from the behavior of the individual.

The condition that the particles which belong to a size group are in direct contact to each other if blown up is equivalent to a uniform distribution of the particles. But that condition is valid only for mixtures where the individuals of a size group are in real direct contact either to each other or to the particles belonging to the larger groups. If they are not, the distances between them will vary according to the laws of chance. In every aggregate mixture which is produced by a mixing machine there will be parts with and without particle interference. From the quantity proportions of the two size groups and their size ratios, we can conclude only to the probable relative extent of the two parts.

There exists no possibility for preventing particle interference in any machine-mixed aggregate, and there is no reason why we should. We know of many excellent gradings which do not comply with the authors' formula. Particle interference is only one phenomenon of many with which the engineer must deal, just as there are, for example, loads, tensions, strains, or ground water, and which he has got under his control with more or less perfection.

It is up to the engineer to investigate the nature and the effects of particle interference. But he can perform that only by tests with whole batches of mixes.

*Technische Versuchs- und Forschungsanstalt Kaprun, Land Salzburg, Austria.

The best way will be to start his investigations with the consideration of only two size groups, and to see what influence is exercised by varying quantity proportions of the constituents and by varying ratios between their diameters. Then he might test the influence of the presence of a third and of more size groups, find out the effects of particle shape and surface. Later he will learn to tell from the grading the quantity of cement he must add to get the concrete he wants. At last he will come to know the methods of designing the grading which yields the best quality of concrete—which means, as Fuller has taught us, that mixture which demands the smallest amount of cement—and methods to adapt the grading of ideal composition to the grading which is produced from his aggregate pit or by his crusher.

The knowledge of the behavior of the individual particle is necessary for the understanding of the laws which govern the multitude. But what the engineer must have is a mix which yields a good and workable concrete in the most economical way. Therefore, he cannot afford to go too far with the improvement of the grading of the pit or crusher run aggregates, but must find a wise compromise between the theoretically best mixture and the aggregate available. He can achieve that end only with the aid of the theory of aggregate gradation based on a thorough understanding of the effects of particle interference, which makes possible an exact calculation of the most economic mixture with an accuracy adequate to the engineering sciences.

By H. N. WALSH*

A question raised in the first paragraph of this paper is whether a certain fluidity of the fresh concrete is necessary in order that Weymouth's theory should apply. The authors answer this question in the affirmative as far as two particular gradings are concerned. It appears, however, and as they say, that study of many more gradings would be required to give a full answer. The mathematical argument in favor of their method based on consideration of the complete grading of solids is quite logical, although it tends to cloud one's mental picture of what is happening as compared with the step-by-step method. But the working out of the evaluation of the grading curve A in Table 1 involves nearly all the stages of the step-by-step process. It would be interesting to see the application of the new method to the derivation of a grading to suit a given cement content and slump. Does it involve less labor than the Weymouth method?

The section dealing with "Interpretation of Results" is confusing at first reading because of a misprint which directs attention to the adjacent Table 2 instead of Table 1. This Table 2, which is relevant to the appendix, is taken from Weymouth's original paper, and the aggregates analyzed in it are interesting in relation to the subject of the paper. Due to having only 7 percent passing the No. 52 sieve, the sand is a little coarse, its fineness modulus being 3.14. The 1½- to 3/16-in. gravel has 25 percent of 1½–¾-in. size, 50 percent of ¾–⅜-in., and 25 percent of ⅜–3/16-in. (No. 4). Excess of the second

*Professor of Civil Engineering, University College, Cork, Ireland.

largest size in a coarse aggregate tends to cause harshness and this combined with a sand lacking in the extreme fines would lead one to expect that the proportion of sand to coarse aggregate would have to be higher than average to give good workability. Thus one can see by inspection that the relatively small proportion of 1½- to ¾-in. stones would be well separated in the concrete. Table 2 shows that the particle interference theory requires a clearance of slightly over 1-in. between the particles of this group. That means their average distance apart center to center would be about 2⅛ in. Are such spacings *always* necessary?

In his 1938 article in *Engineering News-Record* Weymouth stated: "The great advantage of this method is that it insures the greatest workability possible from the available materials for the water-cement ratio and slump chosen." Further on he said: "In architectural concrete maximum workability is imperative." Well, in many engineering structures density, durability, and strength are imperative. Workability may be a secondary consideration; especially where labor rates are low; even for hand-tamped plastic concrete of less than 3-in. slump, gradings containing high proportions of the top sizes can be used economically. The following examples may be of interest. In a certain area the only sand economically available has the grading No. 1 in Table A. In one job this was used with ¾- and ⅜-in. gravels having the gradings No. 2 and 3.

TABLE A—GRADING EXAMPLES

	Reference	Sieve sizes and percentages by weight passing								
		No. 100	No. 52	No. 25	No. 14	No. 7	⅜ in.	¾ in.	¾ in.	1 in.
1	Sand	0.8	17.6	68.7	93.7	100	—	—	—	
2	¾-in. gravel	—	—	—	—	—	0	28.2	100	
3	⅜-in. gravel	0.2	0.4	1.1	3.5	13.1	69.1	98.9	100	
4	Above combined	0.2	5.3	20.7	32.2	31.8	39.6	59.6	100	
5	Another combined aggregate	0.7	6.3	19.8	29.5	33.7	37.3	47.7	82.5	100

These were combined to give grading No. 4 in which the fines of the ⅜-in. gravel help to make the grading of the sand part more normal. Yet the proportion of No. 25–52 size is high. This combined aggregate was used to make concrete containing 630 lb of cement per cu yd and slump not exceeding 3 in. The concrete was easily placed and compacted by hand in the floors and walls of small reinforced concrete reservoirs. The horizontal construction joints in the walls were plane surfaces which were carefully cleaned to expose the coarse aggregate and washed before placing of the next lift. There were no waterstops and no layer of mortar at the bottom of a lift. There was no visible sign of segregation anywhere; the surfaces were perfect and there was no leakage in reservoirs when filled.

In another part of the same area, sand from the same source was used with a nominal ¾-in. crushed stone, all of which passed a 1-in. sieve and less than 12 percent passed a 3/16-in. sieve. These were combined to give grading

No. 5 of Table A. The combined aggregate was used in a concrete containing 630 lb of cement per cu yd and slump of less than 2½ in. to make piles, cantilever retaining walls, and a heavily reinforced rigid frame. The concrete was easily placed and compacted and the finish was good despite the 62.7 percent by weight of crushed stone coarse aggregate. Strengths and densities were high.

In both these cases density, strength, and durability were the important properties, with sufficient workability to make placing and compacting simple and satisfactory with the labor and supervision available. The proportions of these concretes were determined by short and simple calculations based on the sieve analyses and checked by trial mixes.

Where vibration is used for compaction, both water and sand contents may or must be lowered to avoid danger of segregation or to produce high strengths and densities. Various methods of design are available. It is relevant to this discussion to note that in England and France theories assuming little or no clearance between particles of the coarse aggregate have been used, especially in conjunction with gap gradings, to design concretes of high density and strength, apparently with success. It is obvious, therefore, that concretes can be made successfully without the spacings required by the particle interference theory. This is said, not by any means to depreciate Weymouth's work, but to indicate that the application of his method would appear to be limited, and thus to support one of the authors' conclusions.

AUTHORS' CLOSURE

The authors regret the misleading effect of the error in the first paragraph of the section "Interpretation of Results." The error was discovered in the galley proof stage, but unfortunately time and distance conspired to make a cable to the editor arrive too late for the error to be corrected. As pointed out in discussion, the grading analyzed in *Table 1* (not Table 2) is plotted in Fig. 4 as grading A. Table 2 should be read in conjunction with the appendix, in which the relevant statement is ". . . Table 2, which reproduces part of Table 1 of Weymouth's original paper."

Two experiments may be carried out: (1) to show that workability can be changed by varying the gradings of solids while maintaining a constant ratio of water to solids; and (2) to show that workability can be changed by varying the ratio of water to solids while maintaining a constant grading.

Experiment (1) above shows the importance of grading of solids in determining workability, and the paper is concerned only with this aspect. It in no way minimizes the importance either of the ratio of water to solids, or of interparticle forces such as surface tension. The authors therefore agree with Mr. Powers' contention that their analysis has intrinsic limitations, and their qualifications regarding fluidity in the first and last paragraphs of the paper were intended to indicate this.

The authors cannot agree with Mr. Powers when he says that there is "no logical escape from the necessity of including the still smaller and all important particles, the water molecules," and he has given the reasons for not including them. He says: (a) "Water molecules do not pack like ordinary particles," and (b) "if the computation had included water molecules, the weighted size of the cement particles" (and therefore the aggregate) "would also have been negligible." One would therefore be forced to conclude that only water affects workability, and that grading is unimportant. This would conflict with the results of experiment (1) above, and with general experience.

Nevertheless, Mr. Powers' objection to a procedure that treats all solid particles alike is valid. The grading of solids when dry, their grading when moist, and their grading when saturated, will vary, and the variation will be most significant in the finest sizes, where the analysis is most sensitive. This indicates a field for further study, namely, the effects of water content and of compacting effort upon flocculation of the fine particles of a concrete mix. A measure of flocculation which led to a reliable estimate of particle size of cement would be most valuable, as it would allow a more realistic value of particle size to be substituted in Table 1. Hydrometer tests, upon which the cement size quoted in Table 1 is based, cannot be regarded as satisfactory for this purpose.

Professor Walsh has correctly interpreted the authors' qualifications regarding fluidity. It may be, as he remarks, that the step-by-step method appears to give a clearer picture than the analysis shown in Table 1. Any such advantage is only apparent, however, for the step-by-step method, as shown by the appendix and Fig. 7, leads to results which are in conflict with recognized physical laws.

Excess of the second largest size of coarse aggregate decreases the value of D_o and hence also the maximum permissible value of D_a (Fig. 3).

Professor Walsh's remarks upon Weymouth's *Engineering News-Record* article are interesting. It is outside the scope of this paper, but there is nothing incompatible in grading for maximum density and for maximum workability, and it can be shown that the two curves are of the same family, with a high proportion of coarse aggregate.

His Table A shows two gradings with 39.6 and 37.3 percent of fines, and high proportions of No. 25–52 material. These gradings would have high resistance to segregation, and with slumps of 2½ to 3 in. the concrete should be quite workable. The relevant point, of course, is the amount of water required to give this workability.

Mr. Schwanda refers to the way in which the behavior of a large collection of particles may vary from that observed for individual particles. While this is true, there is a statistical quality about the behavior of the collection which allows valid general conclusions to be drawn. There are, indeed, many excellent gradings which do not comply with any one formula. It is suggested that the reason for this is that they are designed for different purposes.

The authors emphasize that the scope of their paper is limited to a consideration of particle interference, and they have not applied it to cover all phases of workability, nor have they applied it to maximum density, or segregation. It is relevant to state, however, that if minimum particle interference gives maximum workability it also gives minimum resistance to segregation. It is impossible, therefore, to consider a grading with maximum workability as a satisfactory practical grading. The authors believe, however, that a fundamental study of each and every factor affecting the design of concrete mixes is necessary before they may be combined to satisfy every practical requirement.

Discussion of a report by ACI Committee 214:

Proposed Recommended Practice for Evaluation of Compression Test Results of Field Concrete*

By G. H. McPHERSON, R. SHALON and R. C. REINITZ, WALTER K. WAGNER, and COMMITTEE

By G. H. McPHERSON†

The committee is to be commended for its efforts to provide generalized instructions for the application of statistical methods to problems facing the industry.

When used to draw inferences, these methods assume "controlled conditions," i.e., that the samples are random from a stable population. When applying these methods, an investigator must design his sampling program to provide clear evaluation of these assumptions. The quality control charts mentioned in the report are excellent tools for investigating the stability of the production process.

Randomization is essential to the drawing of unbiased samples. Some, if not most, technicians have a tendency to sample what appears to be a "good" batch so as to avoid the necessity of explaining low breaks. To obviate this we use a table of random numbers to select the loads from which samples are drawn. The numbers are known only to the concrete engineer and the technician who draws the sample. Samples are drawn in subgroups of three plotted to $\overline{X}$ and R charts and examined for assignable causes before combining the subgroups for analysis of the type the committee recommends.

I believe the committee should include a discussion of these points in connection with the use of Fig. 3 and 5.

Willis and Murray, in discussion‡ of an earlier report by Committee 214, which formed the basis for the proposed recommended practice, present a cogent argument for the standard deviation as opposed to the coefficient of variation. The committee's Table E‡ does not to my mind ". . . indicate a definite correlation between the mean and the standard deviation." From the table alternate hypotheses that s is correlated with age or interaction between age and mean appear tenable.

*ACI JOURNAL, Dec. 1956, *Proc.* V. 53, p. 561. Disc. 53-30 is a part of copyrighted JOURNAL OF THE AMERICAN CONCRETE INSTITUTE, V. 28, No. 12, June 1957, *Proceedings* V. 53.

†Manager, Transit Mix Concrete, Ltd., Honolulu, T. H.

‡Discussion of "Evaluation of Compression Test Results of Field Concrete," ACI JOURNAL, Part 2, Dec. 1956, *Proc.* V. 52, pp. 1173-1181.

By R. SHALON* and R. C. REINITZ†

The interesting report of Committee 214 fulfills the long-felt need for directives which enable full advantage to be taken of controlled concrete, and the committee is to be complimented on the excellent work done.

The recommended practice for the evaluation of the compressive strength of concrete as a result of the strength data obtained in test specimens is based on the assumption that the strengths follow a pattern of normal distribution.

In concrete rated "excellent" or "good" (coefficient of variation $\leqq 15$ percent) the test for normality will, in fact, not reject this hypothesis. In poorly controlled concrete, however, a discrepancy between the observed distribution and the theoretical normal distribution becomes noticeable and a skewness occurs on the side of the higher strengths.[1,2,3] This skewness, and the fact that the strength of concrete is an exponential function of the water-cement ratio have led the writers of this discussion to the assumption of a logarithmic normal distribution of strengths.

This assumption was checked by the writers. They investigated a number of large building projects, independent from each other.[1] The strength of the concrete investigated varied from $v = 14$ percent in the best controlled project to $v = 33$ percent in the poorest one. The test of normality after logarithmic transformation did not lead to rejection of the hypothesis.

Hence instead of calculating f_{cr} for a specified strength f_c'

$$f_{cr} = f_c' \frac{1}{1 - vt}$$

the appropriate equation would be[4]

$$f_{cr} = f_c' \, e^{v(t + v/2)}$$

This means that whenever $t \geqq 1$ and $vt < 1$ the assumption of a normal distribution would call for a higher average strength than required on the basis of logarithmic-normal distribution. The difference is negligible when the coefficient of variation falls below 15 percent and/or when it is required that less than 90 percent of the strengths fall above the specified strength. It is however of considerable practical value for concrete with a high coefficient of variation, as well as whenever only a small percentage of all the strengths is to fall below the specified strength.

A coefficient of variation of 15 percent and above is often met with in Europe and in this country, and, as mentioned by Blanks[5] in the United States it may also be as high as 35.9 percent. Moreover, in many countries—as in England for instance—it is usually required that 99 percent of all strengths fall above the specified strength.

*Professor of Civil Engineering and Head, Building Research Station, Technion, Israel Institute of Technology, Haifa, Israel.
†Research Engineer, Building Research Station, Technion, Israel Institute of Technology, Haifa, Israel.

TABLE A—RATIO OF REQUIRED AVERAGE STRENGTH f_{cr} TO SPECIFIED STRENGTH f_c'

Coefficient of variation, percent	Chances of falling below specified strength									
	1 : 10		1 : 20		1 : 40		1 : 100		1 : 200	
	Norm.	Log-norm.	Norm.	Log. norm.	Norm.	Log-norm.	Norm.	Log-norm.	Norm.	Log-norm.
5	1.068	1.068	1.090	1.087	1.109	1.104	1.132	1.125	1.148	1.139
10	1.147	1.143	1.197	1.185	1.244	1.223	1.303	1.268	1.135	1.300
15	1.238	1.226	1.328	1.294	1.416	1.357	1.536	1.434	1.630	1.488
20	1.345	1.320	1.490	1.418	1.645	1.510	1.870	1.625	2.063	1.708
25	1.472	1.422	1.699	1.557	1.961	1.684	2.389	1.846	2.809	1.965
30	1.625	1.534	1.974	1.713	2.427	1.883	3.309	2.102	4.401	2.265

A greater accuracy in the determination of the distribution is therefore of particular economic value. Besides, the logarithmic-normal distribution lends itself to simple mathematical treatment.

In parallel with Fig. 5 (p. 571) of the report, Fig. A shows the effect of normal and logarithmic-normal distribution on the average strength required.

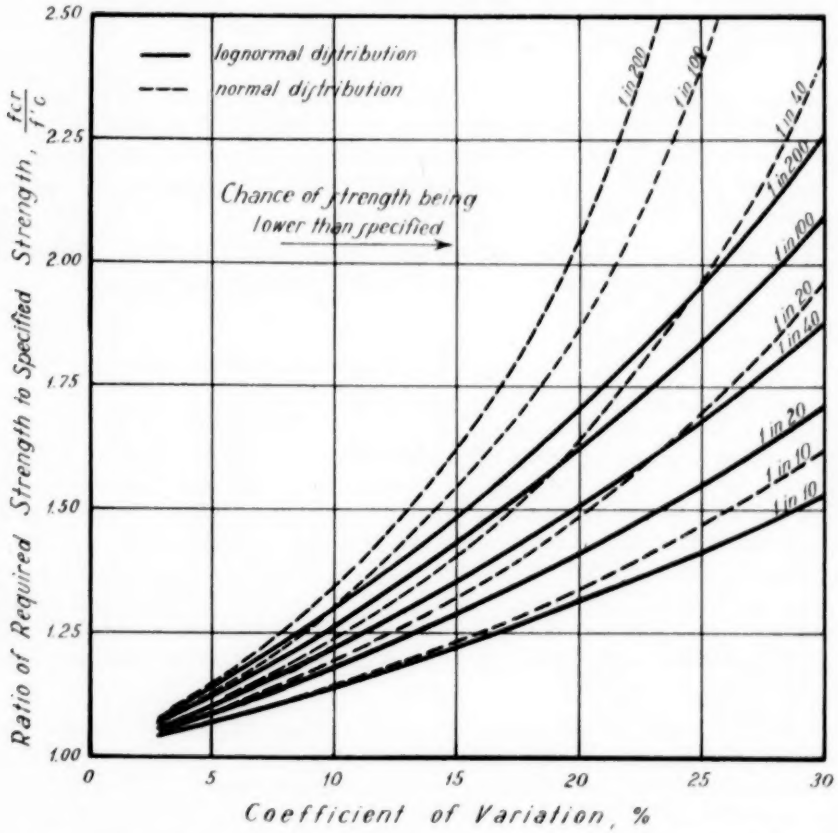


Fig. A—Effect of normal and logarithmic-normal distribution on average strength required

Table A summarizes the results of calculation of the average strength required when either a normal or log-normal distribution is assumed. It is believed that the use of such a table might prove easier to the practical engineer, as it relieves him from calculations.

REFERENCES

1. Shalon, R., and Reinitz, R. C., "Interpretation of Strengths Distribution as a Factor in Quality Control of Concrete," RILEM Symposium, Lisbon, Oct. 1955.

2. Gaede, K., "Anwendung Statistischer Untersuchungen und die Pruefung von Bau-Stoffen," *Der Bauingenieur* (Berlin), No. 40/42, 1942.

3. Ewers, N., "Berechnungsgrundlagen, Sicherheitsfaktor und Materialpruefung im Stahlbeton," *Bauplanung und Bautechnik* (Leipzig), No. 6, June 1953.

4. Hald, A., *Statistical Theory with Engineering Applications*, John Wiley & Sons, Inc., New York, 1952, p. 161.

5. Blanks, R. F., "Concrete Quality Control," *Proceedings*, Building Research Congress, Division 2, London, 1951.

By WALTER K. WAGNER*

Based on experience of recent years this writer recognized that the coefficient of variation was not constant for all classes of concrete within a given operation. In our ready-mixed concrete production we have designed structural mixes for strengths ranging from 1000 psi for lightweight concrete to more than 5000 psi for sand-gravel concrete.

Two years ago an analysis of our test data indicated that our coefficient of variation varied inversely with strength and could be expressed as:

$$V = \frac{K}{S}$$

where V = coefficient of variation in percent
S = compressive strength in psi
K = a constant dependent on the control level

A value of 40,000 was assigned to K based on our field test data. Later it was found that Himsworth† presented data with a similar trend but which gave a K value of 80,000. This expression, $V = K/S$ (an equilateral hyperbola), gave unrealistic values, however, for V for the strength range below 2500 psi. Although our data were too meager to establish a reliable V for this low-strength concrete, it was obvious that 40 percent was unreasonable for the 1000-psi class.

A further study of our data showed that when standard deviations are plotted on a logarithmic scale, as reductions from their respective 28-day strengths, against water-cement ratios, the points appear to lie in a straight line and converge at zero water-cement ratio at the same point of convergence

*Chief Engineer, Albuquerque Gravel Products Co., Albuquerque, N. M.

†Himsworth, F. R., "The Variability of Concrete and Its Effect on Mix Design," *Proceedings*, Institution of Civil Engineers, (London), V. 3, Part 1, No. 2, Mar. 1954, p. 169.

of the 7- and 28-day strength lines. Fig. B shows this plot of four groups of field tests that were made at approximately the same level of control. They were selected here because of a fairly wide range of strength classes. Average strength values for the groups and their respective values for standard deviation and coefficient of variation are given in Table B. The standard deviation lines of Fig. B, which might be called "sigma lines," represent the average strength values reduced by their standard deviations.

It occurs to this writer that this interpretation of the variance of concrete strengths is a logical and realistic one. For a given level of control, such as that of a ready-mixed concrete plant producing a wide range of concrete mixes, it is necessary to develop reliable test data for only one mix to design a complete series of mixes.

The familiar Abrams' expression $S = A/B^x$ is used to denote strength in terms of water-cement ratio, where S = compressive strength at any age, x = water-cement ratio by weight, and A and B are constants. Abrams determined the point of convergence A to be 14,000 psi at zero water-cement ratio.

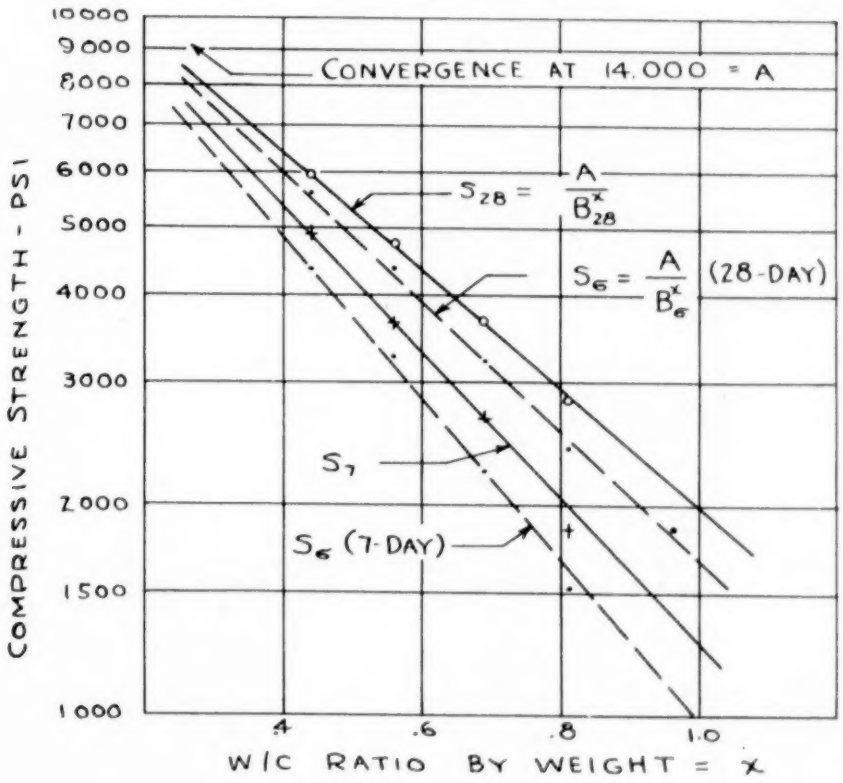


Fig. B—Plot of four groups of field tests

Thus, the 28-day and "sigma" strength lines can be written respectively:

$$S_{28} = \frac{A}{B^x{}_{28}} \quad \text{and} \quad S_\sigma = \frac{A}{B_\sigma{}^x} \text{ (for 28 days)}$$

Subtracting, we get

$$S_{28} - S_\sigma = \frac{A}{B^x{}_{28}}\left[1 - \left(\frac{B_{28}}{B_\sigma}\right)^x\right]$$

but $S_{28} - S_\sigma = \sigma$ = standard deviation, and $A/B^x{}_{28} = S_{28} = \overline{X}$ = average strength, therefore

$$\sigma_{28} = \overline{X}\left[1 - \left(\frac{B_{28}}{B_\sigma}\right)^x\right] \quad \ldots\ldots (1)$$

and since $V = \sigma/\overline{X} \times 100$ the expression reduces to:

$$V = \left[1 - \left(\frac{B_{28}}{B_\sigma}\right)^x\right] 100 \quad \ldots\ldots (2)$$

For the data in Table B, the average values for B_{28} and B_σ are 7.057 and 8.372, respectively, and Eq. (2) for this particular set of data reduces to the simple form:

$$V = [1 - 0.8429^x]\ 100 \quad \ldots\ldots (3)$$

Values for σ and V computed from Eq. (3) are as follows for various 28-day strengths:

S_{28}	W/C	σ	V
2000	0.988	349	17.5
3000	0.784	376	12.5
4000	0.640	414	10.4
5000	0.528	431	8.6
6000	0.436	431	7.2

TABLE B—FIELD STRENGTH TEST DATA FOR READY-MIXED CONCRETE OF WIDE RANGE OF STRENGTHS AT UNIFORM CONTROL LEVEL

	7 days				28 days				
W/C ratio by wt	Average strength, psi S_7	Standard deviation, psi σ	Difference $S_7 - \sigma$, psi S_σ	Coefficient of variation, percent V	Average strength, psi S_{28}	Standard deviation, psi σ	Difference $S_{28} - \sigma$, psi S_σ	Coefficient of variation, percent V	No. of tests
0.81	1830	300	1530	16.5	2830	400	2430	14.3	51
0.69	2660	430	2230	16.3	3670	450	3220	12.2	80
0.56	3660	370	3290	10.1	4700	300	4400	6.4	19
0.44	4930	570	4360	11.6	5960	350	5610	5.8	23

COMMITTEE CLOSURE

The wide interest in the report of Committee 214, as exemplified by the excellent discussions received on the original report and also the proposed standard, is evidence of the need of a standard for evaluation of tests.

The most apparent reason why such a standard has not been produced sooner is the wide difference in thinking between the theoretical statistician and the practical concrete engineer. Committee 214 has endeavored to apply some simple statistical methods in a way which would be readily acceptable and usable by the concrete industry. Assumptions made in the report were based on field tests and experience from a cross section of the concrete industry. It is not surprising that these assumptions do not completely agree with some individual investigations, and some of the refinements in the theory of statistics may have been neglected. It has been the experience of the committee, however, that statisticians themselves do not entirely agree, and it would be difficult to prepare a report completely satisfactory to everyone.

It is the hope of Committee 214 that the proposed standard will not be accepted at face value, but will continue to serve as a stimulus for active discussion and for accumulation of test results and experiences. In this way the report can become more authoritative.